

**PROFESSIONAL DEVELOPMENT
VIA EMAIL WITH
*James Tanton***

LOGARITHMS

**WHAT ARE THEY REALLY?
WHAT ARE THEY GOOD FOR?
WHY DO WE HAVE STUDENTS STUDY THEM?**

HERE IS A START TO A CONVERSATION

Read and think about this passage.
Instructions on what to do next appear at the end.

Opening Question:

CAN YOU JUST TELL ME WHAT A LOGARITHM IS?

Let's not worry about context for the moment or reasons why anyone in their right mind would want to study these things, and just get the very first issue out of the way right away. Let's just figure out what a logarithm is. We can worry about all that other stuff later on.

The mathematics of logarithms is actually, surprisingly, remarkably straightforward. Let's play a game.

Suppose I wrote on a board

$$\text{power}_2(8) = 3$$

and

$$\text{power}_5(25) = 2.$$

Do you think you could guess what is going on? (I am assuming we know about powers of numbers.)

Can you figure out each of these next examples?

$$\text{power}_3(27) = \underline{\hspace{2cm}} \qquad \text{power}_{10}(\text{million}) = \underline{\hspace{2cm}}$$

$$\text{power}_{10}(100) = \underline{\hspace{2cm}} \qquad \text{power}_{73}(1) = \underline{\hspace{2cm}}$$

$$\text{power}_4(16) = \underline{\hspace{2cm}} \qquad \text{power}_{0.01}(1000) = \underline{\hspace{2cm}}$$

$$\text{power}_4(64) = \underline{\hspace{2cm}} \qquad \text{power}_{100}(0.1) = \underline{\hspace{2cm}}$$

$$\text{power}_7\left(\frac{1}{7}\right) = \underline{\hspace{2cm}} \qquad \text{power}_{\sqrt{6}}\left(\frac{1}{36}\right) = \underline{\hspace{2cm}}$$

$$\text{power}_2(\sqrt{2}) = \underline{\hspace{2cm}} \qquad \text{power}_1(5) = \underline{\hspace{2cm}}$$

$$\text{power}_{\frac{1}{3}}(9) = \underline{\hspace{2cm}}$$

The answers column-wise are: 3, 2, 2, 3, -1, 1/2, -2 and then 6, 0, -3/2, -1/2, -4, and impossible! (The last few in the second column are tricky!)

Okay. We're done. We've just done logarithms!

Logarithms are just powers. But for very quirky historical reasons people don't use the word "power" as they should, but instead use the really scary made up word *logarithm*, shortened to just *log*. (There was this fellow by the name of Napier who invented these things. He was saving all of global science from a very annoying basic problem and did a great thing for the world by inventing these things. But no one realized at the time that what he was doing were just powers!)

$$\begin{array}{ll} \log_{\text{power}_3}(27) = \underline{3} & \log_{\text{power}_{10}}(\text{million}) = \underline{6} \\ \log_{\text{power}_{10}}(100) = \underline{2} & \log_{\text{power}_{73}}(1) = \underline{0} \\ \log_{\text{power}_4}(16) = \underline{2} & \log_{\text{power}_{0.01}}(1000) = \underline{-3/2} \\ \log_{\text{power}_4}(64) = \underline{3} & \log_{\text{power}_{100}}(0.1) = \underline{-1/2} \\ \log_{\text{power}_7}\left(\frac{1}{7}\right) = \underline{-1} & \log_{\text{power}_{\sqrt{6}}}\left(\frac{1}{36}\right) = \underline{-4} \\ \log_{\text{power}_2}(\sqrt{2}) = \underline{1/2} & \log_{\text{power}_1}(5) = \underline{\text{impossible}} \\ \log_{\text{power}_{\frac{1}{3}}}(9) = \underline{-2} & \end{array}$$

The point here is that **whenever you see the word *log* just think the word *power*:**

$$\log_b(x)$$

is simply

$$\text{power}_b(x),$$

the power of b that gives the answer x .

WHAT ARE YOUR NATURAL NEXT QUESTIONS IN RESPONSE TO THIS PASSAGE?

Write them here.

Here are top questions from other people who looked at the opening passage.

WHY IS THE DEFINITION OF LOGARITHM IN THE TEXTBOOK SO HORRIBLE?

WHO WAS NAPIER? WHAT BIG PROBLEM WAS HE TRYING TO SOLVE?

I DON'T GET IT. WHAT ARE POWERS?

AREN'T EARTHQUAKES MEASURED WITH LOGARITHMS OR SOMETHING?

OKAY. I GET IT. WHY SHOULD I CARE ABOUT LOGARITHMS?

Select two questions (from your list and the list above) and email them to *JAMES TANTON* at tanton.math@gmail.com.

Of all the questions he'll receive, he'll provide content for the top few that seem most common and/or most pressing.

We'll then repeat this process, building up a "knowledge map" of logarithms, their mathematics and why we allegedly want students to care about them.