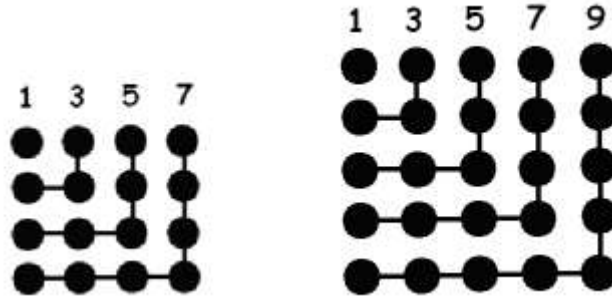


WITHOUT WORDS

Mathematical Puzzles to Confound and Delight



WW 4: SOLUTION



We see that the sum of first four odd numbers, $1 + 3 + 5 + 7$, lies in the picture of a four-by-four array of dots, and so must have sum $4 \times 4 = 16$, the total count of dots in that array. The sum of the first five odd numbers $1 + 3 + 5 + 7 + 9$ sits in a five-by-five array and so has sum $5^2 = 5 \times 5 = 25$.

In general, the sum of the first N odd numbers must be $N \times N$. We get:

$$1 + 3 + 5 = 3^2 = 9$$

$$1 + 3 + 5 + 7 + 9 + 11 = 6^2 = 36$$

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$$

This is astoundingly powerful. For example, we can say without a lick of arithmetic that the sum of the first 1000 odd numbers is 1000^2 , a million. (What is the 1000th odd number?)

In general, the sum of the first N odd numbers is N^2 .

If we add 1 to each term in the sum of the first ten odd numbers

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2 = 100$$

$$+1 +1 +1 +1 +1 +1 +1 +1 +1 +1 \quad +10$$

we get the sum of the first ten even numbers:

$$2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20 = 100 + 10 = 110.$$

In general, the sum of the first N even numbers is $N^2 + N$.

Challenge: Draw a 10-by-11 array of dots. Can you see the sum $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20$ in the picture? (Again look at L shapes.)

Divide the sum of the first ten even numbers by two and get the sum of just the first ten numbers.

$$(2 + 4 + 6 + 8 + 10 + 12 + 14 + 16 + 18 + 20) \div 2 = 110 \div 2$$
$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55 .$$

In general, the sum of the first N numbers is the sum of the first N even numbers divided by two, and so:

$$1 + 2 + 3 + \dots + N = \frac{N^2 + N}{2} .$$

This is a famous school book formula just derived from looking at a grid of dots. See WW 15 for another – even easier – approach using dots.

See also www.jamestanton.com/?p=1006 .