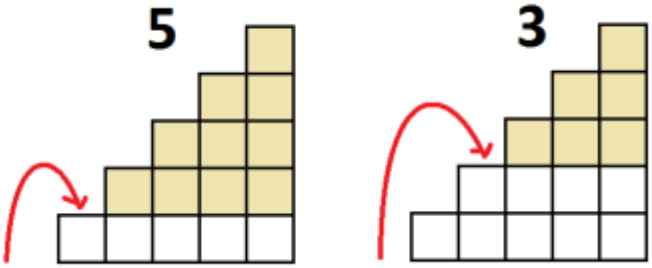
WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

WW 19: SOLUTION

This puzzle is counting the number of ways to walk up a set of stairs one or two steps at a time.

Consider the five-high staircase. If the first step we take is a single step, then we are left to climb a set of four steps and there 5 ways to finish this climb. If, on the other hand, the first step we take is a double step, we are left with a set of three steps, which can be completed 3 ways. There are thus $5+3=8$ ways to climb up a set of five stairs.



The same reasoning shows that there are $8+5=13$ ways to climb a set of six steps.

In this we generate the sequence of numbers 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, … with each term after the second the sum of the two previous terms. This is a very famous sequence of numbers called the *Fibonacci numbers*.

Taking it Further: Count the number of ways to climb up a set of steps taking one, two, or three steps at a time. What sequence of numbers to these counts generate?