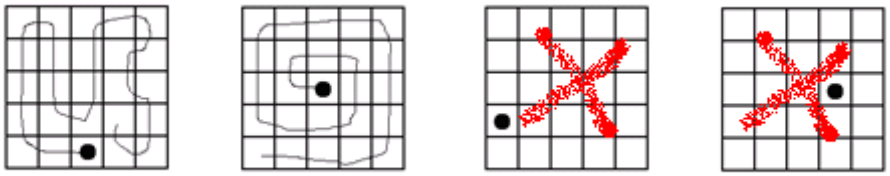
WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

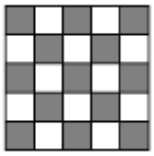
WW 17: SOLUTION



There are multiple ways to solve each of the first two puzzles. The second two puzzles, you might suspect, cannot be solved.

It is easy to prove that something can be done (just do it!) but it is a different matter to prove that something really is impossible.

To make headway on this, colour the grid cells black and white in a checkerboard design. This picture has 13 black squares and 12 white squares.



Now any path that moves horizontally and vertically from square to square must alternately move from a black square to a white square and from a white square to a black square. We can record a sequence of moves then as a sequence of letters:

BWBWBWBWBW…. or WBWBWBWBWB….

But a sequence starting with the letter W will run out of Ws before it runs out of Bs. (There are only 12 Ws but 13 Bs.) Thus a sequence of moves that starts on a white cell will never be able to reach every cell of the board. As the final two diagrams start on a white cell, there can be no path that fills each board.

CHALLENGE: What can you say about path walking on a 6×6 grid of squares? Which cells are permissible starting cells? What about a 6×5 grid of squares?

Comment: Mathematicians use the word *parity* to describe systems or objects that can be in one of two states: up/down, clockwise/counter-clockwise, even/odd, black/white, on/off, for example. To learn more about the mathematics of parity, see Chapter 5 of *THINKING MATHEMATICS! Vol 1: Arithmetic = Gateway to All.* (http://www.lulu.com/shop/james-tanton/thinking-mathematics-1-arithmeticgateway-to-all/ebook/product-17511272.html)

See also WW2, as well as my essay “What made me a mathematician” www.jamestanton.com/?attachment_id=917.