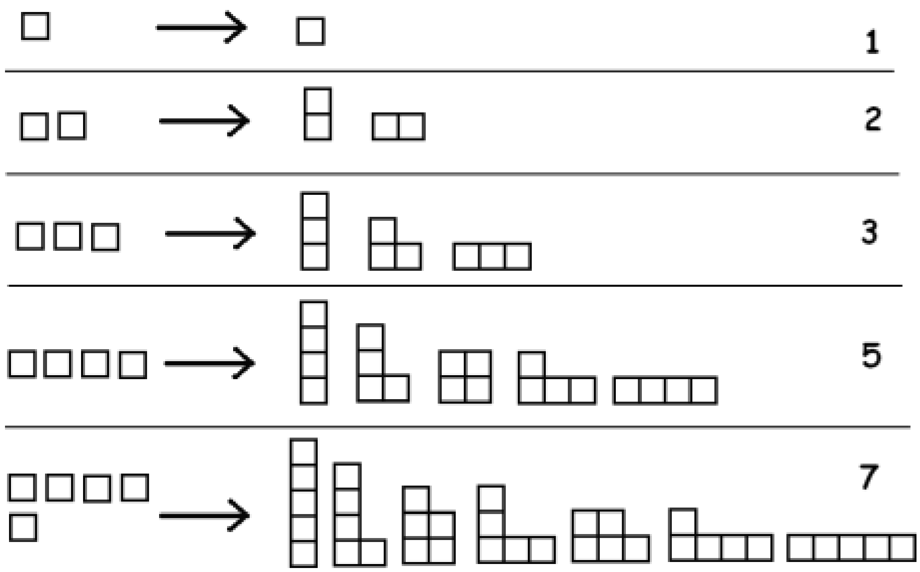
MORE WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

MWW 2: SOLUTION



Each row shows that number of ways to express a given number as a sum (with terms in non-increasing order). For example, the bottom row of the diagram shows that 5 can be written as a sum seven different ways:

$$5 \quad 4+1 \quad 3+2 \quad 3+1+1 \quad 2+2+1 \quad 2+1+1+1 \quad 1+1+1+1+1$$

There is just one way to write 1 as a (somewhat trivial) sum, two ways to write 2, three ways to write 3, and five ways to write 4.

If we let $P(N)$ denote the number of ways to write N as such a sum we have:

$P(1)=1,\ P(2)=2,\ P(3)=3,\ P(4)=5$, and $P(5)=7$.

One checks there are eleven ways to write the number 6 as such a sum. Thus $P(6)=11$.

It seems these count match the prime numbers. But this is just a coincidence. We have $P(7)=15$ and $P(8)=22$.

The sequence continues: $P(9)=30$, $P(10)=42$, $P(11)=56$, and $P(12)=77$.

The numbers $P(N)$ are called the *partition numbers* and no one on this planet currently knows a formula for them.

Famous Indian mathematician Srinivasa Ramanjuan (1887 – 1920) guessed that $P(N)$ is well approximated by the complicated formula

$$P(N) \approx \frac{1}{4\pi\sqrt{3}} e^{\pi\sqrt{\frac{2N}{3}}}$$

(with the approximation becoming more and more exact for larger and larger values of N), which he later proved correct with his colleague G. H. Hardy.

The search for an exact formula continues to this day.