

MORE WITHOUT WORDS

Mathematical Puzzles to Confound and Delight



MWW 13: SOLUTION

For a row of five blocks there are 14 ways to draw arches so that each arch connects two objects - either two blocks, or a block and another arch, or two arches – and so that each block is at the base of some arch.

Either ...

Draw an arch from the first block and connect it to all 5 pictures of arches on four blocks. (5 ways)

Draw an arch on the first two blocks and connect it to all 2 picture of arches on three blocks. (2 ways)

Draw all 2 pictures of arches on the first three block and connect them to an arch on the remaining two blocks. (2 ways)

Draw all 5 pictures of arches on the first four blocks and connect them to the last block. (5 ways)



The same analysis shows that there are $14 + 1 \times 5 + 2 \times 2 + 5 \times 1 + 14 = 42$ ways to construct arches on six blocks.

There are $42 + 1 \times 14 + 2 \times 5 + 5 \times 2 + 14 \times 1 + 42 = 132$ ways to construct arches on seven blocks.

We have the sequence 1, 2, 5, 14, 42, 132, If we put an extra one at the beginning, then the next number in the sequence is generated by writing the portion one has so far both forwards and backwards, multiplying aligned terms, and adding.

$$\begin{array}{ccccccc}
 1 & 1 & 2 & 5 & 14 & 42 & 132 \\
 132 & 42 & 14 & 5 & 2 & 1 & 1
 \end{array}$$

There are $1 \times 132 + 1 \times 42 + 2 \times 14 + 5 \times 5 + 14 \times 2 + 42 \times 1 + 132 \times 1 = 429$ ways to construct arches on eight blocks.

INTERNET RESEARCH: Look up the Catalan Numbers.