

# Curriculum Inspirations

Inspiring students with rich content from the  
MAA American Mathematics Competitions



## Curriculum Burst 18: Sums of Powers of Ten

By Dr. James Tanton, MAA Mathematician in Residence

Consider the set of numbers  $\{1, 10, 10^2, 10^3, \dots, 10^{10}\}$ . The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

**SOURCE:** This is question # 10 from the 2011 MAA AMC 10b Competition.

### QUICK STATS:

#### MAA AMC GRADE LEVEL

This question is appropriate for the 10<sup>th</sup> grade level.

#### MATHEMATICAL TOPICS

Finite geometric sums.

#### COMMON CORE STATE STANDARDS

**A-SSE.4:** Derive the formula for the sum of a finite geometric series (when the ratio is not 1), and use the formula to solve problems.

#### MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

#### PROBLEM SOLVING STRATEGIES

ESSAY 1: **ENGAGE IN SUCCESSFUL FLAILING**

## THE PROBLEM-SOLVING PROCESS:

As per usual ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Alright. I recognize that  $10^{10}$  is the largest number in this list. Are there really another ten elements in that set? Apart from  $10^{10}$ , we have 1, and 10, and  $10^2$ , and  $10^3$ , and all the way up to  $10^9$ . Okay, that is indeed ten more numbers.

Alright, what's the question?

We want the ratio of the largest number to the sum of the remaining ten numbers. (Actually, we want something about which integer it is closest to. That sounds too scary to think about just yet.) Well, here's the ratio we want:

$$\frac{10^{10}}{1+10+10^2+10^3+\cdots+10^9}$$

Heavens!

Let me just try things and hope for the best. Maybe something will come out of it.

The numerator is 10,000,000,000. (That's 1 followed by ten 0s.) Probably not helpful.

The denominator is:

$$1+10+100+1000+\cdots+1,000,000,000$$

and that is

$$1,111,111,111$$

a string of ten 1s. Still probably not helpful.

The ratio we want to consider is:

$$\frac{10,000,000,000}{1,111,111,111}$$

Is that helpful?

Actually, we are looking for the integer  $\frac{10,000,000,000}{1,111,111,111}$  is closest to. Hmm.

How many multiples of 1,111,111,111 "fit into" 10,000,000,000?

I can't help but think:

$$9 \times 1,111,111,111 = 9,999,999,999$$

which is one less than ten billion. Oooh!

$$\frac{10,000,000,000}{1,111,111,111} = \frac{9,999,999,999+1}{1,111,111,111} = 9 + \frac{1}{1,111,111,111}$$

The ratio is really really close to 9. We're done!

**Extensions:**

- a) Is  $\frac{10^N}{1+10+10^2+\cdots+10^{N-1}}$  always close to 9?
- b) If  $b$  is a positive integer, is it possible to say what integer  $\frac{b^N}{1+b+b^2+\cdots+b^{N-1}}$  is closest to? (Do you need to assume that  $N$  is above a certain value?)

*Curriculum Inspirations is brought to you by the Mathematical Association of America, MAA American Mathematics Competitions, and Akamai.*