# Curriculum Burst 6: Areas in Triangles

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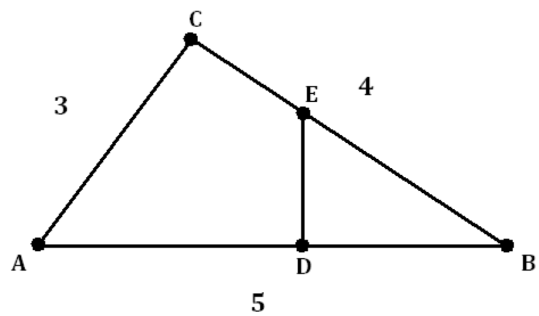
The area of $\Delta EBD$ is one third of the area of $3-4-5$ $\Delta ABC$. Segment $DE$ perpendicular to segment $AB$. What is $BD$?




**SOURCE:** This is question # 9 from the 2011 MAA AMC 10b Competition.

## QUICK STATS:

### MAA AMC GRADE LEVEL

This question is appropriate for the $10^{th}$ grade level.

### MATHEMATICAL TOPICS

Geometry: Similar Triangles, Scale.

### COMMON CORE STATE STANDARDS

**G-SRT.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

### MATHEMATICAL PRACTICE STANDARDS

**MP1** Make sense of problems and persevere in solving them.
**MP2** Reason abstractly and quantitatively.
**MP3** Construct viable arguments and critique the reasoning of others.

### PROBLEM SOLVING STRATEGIES

ESSAY 1: **SUCCESSFUL FLAILING: LIST WHAT YOU KNOW**

## THE PROBLEM-SOLVING PROCESS:

A vital first step:

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STEP 1: Read the question, have an
emotional reaction to it, take a deep
breath, and then reread the question.
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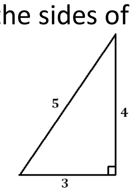
This question has a familiar feel to it. It looks like an exercise from a textbook on geometry (and I have done plenty of geometry textbook questions!). AND it involves a $3-4-5$ triangle, the classic example of a right triangle. Even though I don't see right away what to do, the question doesn't feel too scary.

Let me start by listing what I know about $3-4-5$ triangles.

- A $3-4-5$ triangle is a right triangle with right angle between the sides of lengths $3$ and $4$.




- We have $3^2+4^2=5^2$.
- The area of the triangle is $\frac{1}{2}\times 3\times 4=6$.

This means that the area of the small triangle, $\Delta EBD$, is $2$.

What else do I know?

This question really does look like an exercise from a geometry book. I have two triangles in the picture so it seems natural then to ask: *Are they similar triangles?*

Well, $\Delta ABC$ and $\Delta EBD$ both share the angle at $B$. They have at least one angle in common. Actually, the segment $DE$ is perpendicular to the base of $\Delta ABC$ and so $\Delta EBD$ also contains a right angle, like $\Delta ABC$. Okay, by the $AA$ principle, $\Delta ABC$ and $\Delta EBD$ are indeed similar.

What do I know about similar triangles?

- All angles match exactly.
- All sides match up to some scale factor $k$.

But this question is about areas. Do I know anything about similar triangles and area?

- If one scales a figure by a factor $k$, its area changes as $k^2$.

Okay! The small triangle has area $2$ and the larger, similar triangle has area $6$. This tells us that $k^2=3$ and so the scale factor between the two triangles is $k=\sqrt{3}$.

Umm. What was the question? What are we being asked to do?

*What is $BD$?*

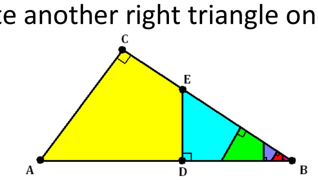
Side $BD$ in the small triangle matches with which side in the large triangle? Well, $BD$ lies between $\angle B$ and the right angle of $\Delta EBD$. The side between $\angle B$ and the right angle in $\Delta ABC$ is $BC$, of length $4$.

Super! So $BD\times\sqrt{3}=4$ giving $BD=\frac{4}{\sqrt{3}}$.

We're done!

**Comment:** In the contest itself, this answer does not appear among the multiple choice options given. Students are expected to recognize this number as $\frac{4\sqrt{3}}{3}$. See the video www.jamestanton.com/?p=513 for a discussion on the strange reasons why school curricula still insist on rationalizing the denominator.

**Extension:** Suppose we repeat this construction infinitely often: Draw a perpendicular line segment in each right triangle to create another right triangle one-third the area.




Find the areas of each of the colored pieces shown. Their (infinite) sum adds to $6$. (Why?) Write down that infinite sum!

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