

# Curriculum Inspirations

Inspiring students with rich content from the  
MAA American Mathematics Competitions



## Curriculum Burst 1: Two Trees

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The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio 3 : 4 .  
In feet, how tall is the taller tree?

**SOURCE:** This is question # 11 from the 2010 MAA AMC 8 Competition.

### QUICK STATS:

#### MAA AMC GRADE LEVEL

This question is appropriate for the 8<sup>th</sup> grade level.

#### MATHEMATICAL TOPICS

Ratio and Proportion

#### COMMON CORE STATE STANDARDS

- 6.RP.1** Understand the concept of a ratio and use ratio language to describe a ration relationship between two quantities.
- 7.RP.2** Recognize and represent proportional relationships between quantities.
- 7.RP.3** Use proportional relationships to solve multistep ratio and percent problems.

#### MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.

#### PROBLEM SOLVING STRATEGY

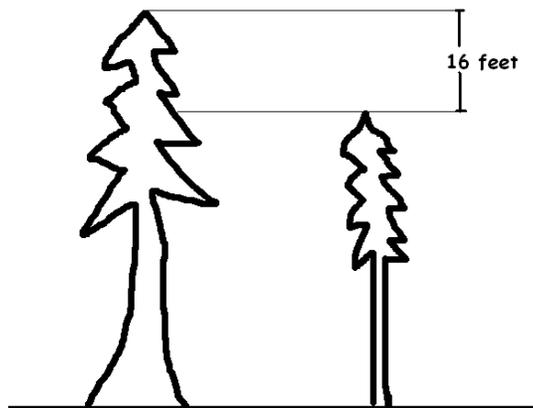
ESSAY 4: **DRAW A PICTURE**

## THE PROBLEM-SOLVING PROCESS:

As always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I don't feel I can really "take in" this question unless I draw a picture. Here are two trees, one 16 feet taller than the other.



Okay, the question now feels manageable. Except, as I read it again, the sentence:

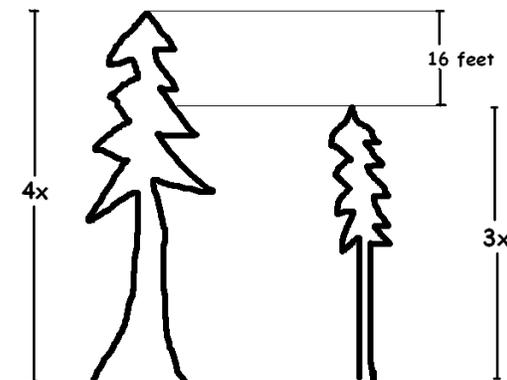
*The heights of the two trees are in the ratio 3 : 4 .*

is cryptic. Does it mean one tree is three feet tall and the other is four feet tall? No.

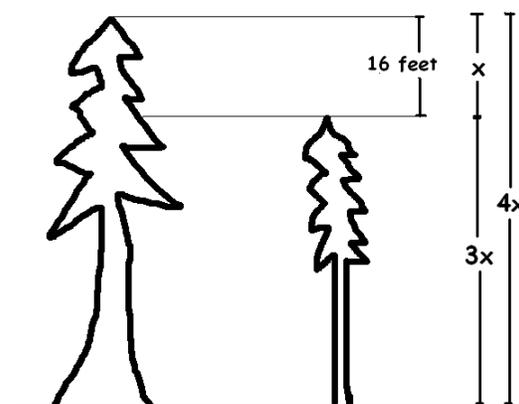
Hmm. What does the "ratio 3 : 4" mean?

To get a feel for it, let's ask: What are some possible tree-heights that would be in a 3 : 4 ratio? Well, 30 feet and 40 feet are in a 3 : 4 ratio. So are 60 feet and 80 feet. And 120 feet and 160 feet.

Okay. Heights being in a 3 : 4 ratio means one tree is of height  $3x$  and the other of height  $4x$  for some number  $x$ . Let's mark this on the diagram.



Oh. Look at the picture! It is now obvious that  $x$  is 16!



So the tallest tree is  $4x = 4 \times 16 = 64$  feet tall!

**Extension:** Would the question be significantly harder if were told that one tree is still 16 feet taller than the other but that their heights now come in a 31 : 32 ratio? How about a 117 : 201 ratio? (These might lead to very tall trees!)

*Curriculum Inspirations is brought to you by the Mathematical Association of America, MAA American Mathematics Competitions, and Akamai.*