TANTON'S TAKE ON …

WHO CHOSE THOSE NUMBERS?

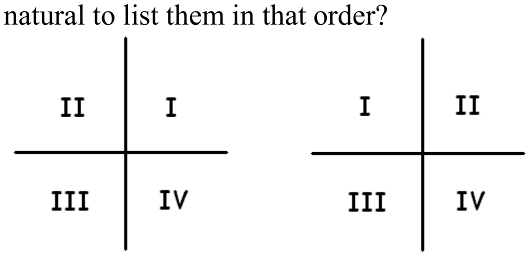
CURRICULUM TIDBITS FOR THE MATHEMATICS CLASSROOM

FEBRUARY 2013

1. Why do we base our number system on ten - ones, tens, hundreds (tens of tens), thousands (tens of hundreds), and so on? Who chose the number 10?

2. We do we say there are 360° degrees in a circle? Who chose the number three-hundred-and-sixty? Why do mathematicians, on the other hand, choose to work with the number 2π when thinking one full turn?

3. Why do we number the four quadrants of the plane a funny way? We read left to right, from up to down. Would it not be more natural to list them in that order?



Weird! More natural?

4. Why are these questions, and others like them, not usually addressed in our curriculum?

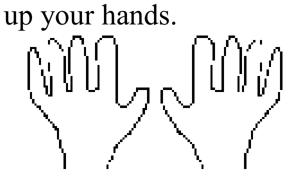
This month's essay is very much about being human. It's math!

The first three questions have wonderfully human stories behind them – accessible and natural to one and all. (I do not know the answer to the fourth question!) Let's embed the stories in the work we do with our students! Enjoy telling them to classes.

Are you wearing a watch with Roman numerals? Are you sitting near a grandfather clock or near a town clock tower? What do you notice about the number four on the clock face? It does not appear as IV !!
Do you know why?

WHO CHOSE BASE TEN?

The answer is … we all did! It is our biology that naturally makes us humans think "ten." Just hold up your hands.

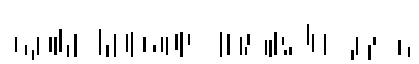


Actually… there are a number of cultures that think "20" instead, or as well. (Why twenty?) There are tinges of base twenty even in our western culture. For example:

How do the French say the number 81? *Quatre-vingt-un* which literally translates as "four twenties and one."

How does the Gettysburg address begin? *Four score and seven years ago* .. And what is a "score"? Twenty years!

Comment: In 1937 archeologists in Moravia uncovered the radius bone of a wolf dated possibly 30,000 B.C.E. on which 55 notches were carved with 25 notches appearing in groups of five.



This strongly suggests that Paleolithic man was counting (counting what? deer? days?) and, moreover, that she eventually realized that grouping notches into groups of five made matters easier to read. (And you can guess why the number five!)

Question: MARTIAN COUNTING
I happen to know that Martians have two hands with four fingers on each hand. On which number do Martians base their counting system? How does their version of the Gettysburg address begin?

WHO CHOSE THE NUMBER 360 FOR THE NUMBER OF DEGRESS IN A CIRCLE?

Mathematics is an intensely human enterprise and its development is steeped in humanness. Let's nut our way through this question, keeping our humanness in mind to see where "360" could have possibly arisen.

What in our human experience makes us naturally think "cycles"?

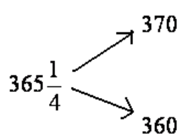
Answer: The year and its seasons.

The ancient Babylonians of around 1500 B.C.E. were fully aware that there are $365\frac{1}{4}$ days in a year, so it seems natural to associate this number with "one full cycle."

Would you like to do math with this number $365\ 1/4$?

Of course, not! So what is the natural thing to do – round this number to something more manageable.

It seems natural to round an awkward number to the nearest ten, in this case to round $365\frac{1}{4}$ *to* 370. *But why did the Babylonians choose to round down to* 360 *instead?*



Since all arithmetic and calculation was down by hand in ancient (and fairly recent) times, it would be most helpful to work with a number that divides into parts easily.

Divisible?	360	370
	2	2
	3	~~3~~
	4	~~4~~
	5	5
	6	~~6~~
	~~7~~	~~7~~
	8	~~8~~
	9	~~9~~
	10	10
	~~11~~	~~11~~
	12	~~12~~

We see that 360 is a much friendlier number when it comes to arithmetic.

Question: Why do you think the Babylonians chose to round to the nearest ten and not to 365, the nearest five?

Comment: The number 360, associated with the passing of a year, become associated with all matters of time. As three-hundred-and-sixty, in and of itself, is a large, unwieldy number it was natural to break this number down and think of if it six groups of 60. This led the Babylonians to create a base-sixty number system. And from this we have our units of time based on the number sixty: 60 seconds in a minute, 60 minutes in an hour. (Why the number 24 for the number of hours in a day?)

Question: MARTIAN CIRCLES
A Martian day is called a Sol (it is 24 hours and 37 minutes long) and there are 668.6 Sols in a Martian year.
Ancient Martians naturally came to associate the number "668.6" with one full cycle. But that is an awkward number for arithmetic. What do you think Martians eventually settled on for the number of degrees in their circles?

WHY DON'T MATHEMATICIANS USE THE NUMBER 360?

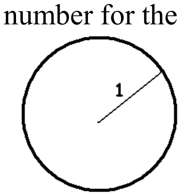
The number 360 comes from our human experience – us humans who happen to be on this particular planet orbiting this particular Sun. It has nothing to do with mathematics itself and so it is not surprising that as we progress with our mathematical studies, the number 360 starts to feel strange and awkward to the subject.
So... being common-sense humans (even as mathematicians!) let's ask:

What is the most natural mathematical object to associate with the notion of "one full cycle"?

Answer: A circle.

And to make life as straightforward as possible, what would be the simplest circle to work with?

A circle of radius 1 seems simplest! (Why choose a hard number for the radius?)



And if I were to walk once around this circle, how far would I actually walk?

According to the formula $2\pi r$ with $r = 1$ we walk a distance of 2π .

So let's now associate the number "2π" with one full turn, the distance one physically walks to go once around the simplest circle. (So now half a turn, 180°, is deemed π, and a quarter of a turn, 90°, is $\frac{\pi}{2}$, and so on.) This is a natural mathematical construction not at all locked to the fact we are Earthlings living on this particular planet.

> I bet Martians too have come up with the number 2π to represent one full turn. Do you see why I think this?
>
> **Mathematics transcends our humanness!**

Question: For those who know calculus, show that the derivative of $\sin x$ is $\frac{\pi}{180}\cos x$ if x is given in degrees.
(Calculus would be absolutely littered π s and 180 s if we insisted on using degrees!)

WHY DO WE NUMBER THE QUADRANTS IN A FUNNY WAY?

What are the big questions of mankind?

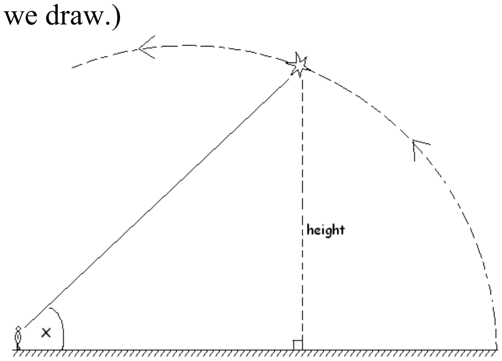
Who are we? Why are we here? Where are we? What is this universe we live in?

Ancient scholars studied the motion of the stars, the moon, and the Sun as a natural part of understanding the world in which we seem to be placed. So much of early mathematics was motivated by astronomy.

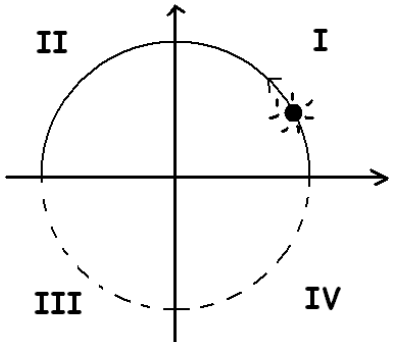
It seems natural to assume that the motion of the stars across the night sky, and the motion of the Sun across the day sky, is more or less along circular arcs. We of course wonder:

How high are these things?

The trouble is we can't climb a ladder to measure their altitudes with a rope. The only measurement we can make is the angle of elevation to these objects from our place on the ground. And so the study of "circle-ometry" was born: understanding the locations of points on a circle given their angles of elevation. (This study later became known as *trigonometry* as it seemed easier to study the triangles we see in the diagrams we draw.)



At some point in our history it became the convention to draw maps with north pointing upwards and east to the right. So in a picture of the motion of the Sun on a coordinate plane, it was natural to place the rise of the Sun on the positive x-axis (to the east) and the setting of the Sun on the negative x-axis (to the west).



And what is the first quadrant the Sun moves through? The top-right quadrant. And where does it go next? The top-left quadrant. And so on. This explains the numbering we use.

Comment: In these diagrams the motion of the Sun is counter-clockwise. This is why mathematicians to this day consider counter-clockwise the positive direction of circular motion, despite, in everyday life, we constantly think and see "clockwise." (Look at the hands on any clock!)

Question: Which direction does the shadow of the *gnomon* (the L-shape piece) in the middle of a sundial move around its face of as the hours pass? Does this explain why we associate the clockwise direction with matters of time?

COOL WORD: *Widdershins* is a 16^{th}-century term for "counter-clockwise." (I use this word as often as I can!) What is the 16^{th}-century companion word for "clockwise"?

Question: MARTIAN QUADRANTS
Is there any reason to expect Martians to label their quadrants in the order we follow?

Question: The science-fiction TV series *Voyager* was about a ship lost in the "delta quadrant." The two-dimensional plane is divided into four quadrants. Is it right to think of three-dimensional space as divided into quadrants as well? If you are in the "delta quadrant" (or should that be "delta octant" ?!) could you still be close to home?

SOME VIDEOS:

Here is a sampler of videos detailing some more humanness behind some familiar mathematical topics.

On 360° in a circle and radian measure:
www.jamestanton.com/?p=633

"Logarithms" a weird (and scary!) name for something simple. Why?
www.jamestanton.com/?p=553

Why is $e \approx 2.71828459045...$ considered the natural number to use for exponents and logarithms?
www.jamestanton.com/?p=611

(And why is the "e" one learns in calculus the same "e" one learns in pre-calculus with regard to compound interest.
www.jamestanton.com?/p=614)

The danger of square roots (because of their history)
www.jamestanton.com/?p=560

Why is *tangent* in trigonometry called "tangent"?
www.jamestanton.com?/p=640

Why on Earth would anyone want that sort of formula for standard deviation?
www.jamestanton.com/?p=709

Why do we flip a graph across a diagonal line for drawing the inverse function?
www.jamestanton.com//?p=764

Why is 0! equal to 1?
www.jamestanton.com?/p=590

How "sine" got its name – a curious story of mistranslation. COMING!
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