CURRICULUM INSPIRATIONS: www.maa.org/ci

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TANTON'S TAKE ON …

TEACHING ALGORITHMS

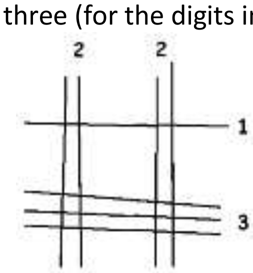
MARCH 2015

Let me teach you an (admittedly unusual) algorithm for performing long multiplication.

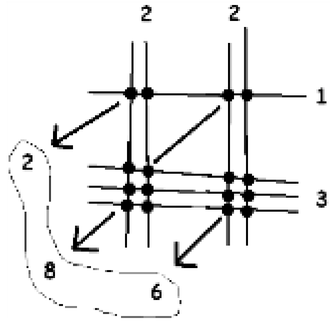
Imagine that this was part of a rote, unenlightened curriculum and that your role as a student is to learn the steps of this algorithm and prove to me that you can repeat the algorithm, with speed (is that demonstrating "fluency"?)—first in a worksheet of 30 practice problems and then in a timed quiz of 15 problems. If you do well, I'll give you a check plus, and then we'll move on to the next topic.

HOW TO DO LONG MULTIPLICATION:
We'll illustrate the method with an example. Let's compute 22×13.

To do this, draw two sets of vertical lines, the left set containing two lines and the right set two lines (for the digits in 22) and two sets of horizontal lines, the upper set containing one line and the lower set three (for the digits in 13).

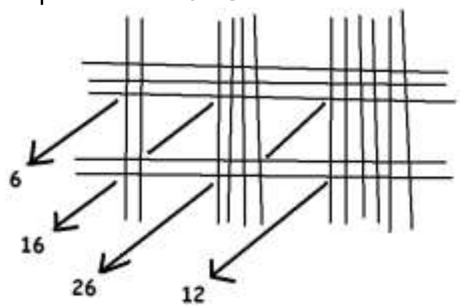


There are four sets of intersection points. Count the number of intersections in each and add the results diagonally as shown:



The answer 286 appears.

There is one caveat as illustrated by the computation 246×32:



Although the answer 6 thousands, 16 hundreds, 26 tens, and 12 ones is absolutely correct, one needs to carry digits and translate this as 7,872.

a) Compute 131×122 via this method.
b) Compute 54×1332 via this method.

and then compute the remaining 28 problems on the worksheet

Surely the burning question in your mind right now is:

WHY DOES THIS METHOD WORK?

I mentioned in my February 2015 Curriculum Essay that even unthinking methodical algorithms can invite valuable problem-solving thinking and practice.

Let me give you a problem:

c) How best should one compute 102×30054 via this method?

But it is true that the burning question at hand is:

d) Why does the method work in general?

An unenlightened curriculum will have the students practice parts a) and b), tell students how to deal with the challenge of c) (rather than let students nut it out for themselves), and likely not address part d).

Now think about the standard algorithm for long multiplication with fresh eyes. It is somewhat strange too:

$$\begin{array}{r} 37 \\ \times\ 23 \\ \hline 21 \\ 90 \\ 140 \\ 600 \\ \hline 851 \end{array}$$

Some weird features:

Start from the right and work left. (We are taught to read left to right, so why not left to right now? Are all algorithms in mathematics in reverse order? Long division?)

Do ordinary multiplication of the single digits, but insert zeros along the way.

Complete the multiplication by performing ***addition****!*

The natural burning question is:

What is the long multiplication algorithm really doing? Why does it work?

I invite you to chat with my teaching colleagues in Grande Prairie, Alberta. They have been mulling deeply on these sorts of questions for the past few months, trying to really see the curriculum for what it is through the eyes of the students. After all, it is the students' perspective of the experience that really counts in the end determining their mathematical success.

tanton.math@gmail.com