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TANTON'S TAKE ON …

MATH IS A LANGUAGE

MAY 201

People often cite "math is a language," usually as a statement of delight and wonder, if not awe, about the applicability of mathematics in describing the universe. I believe this claim stems from Galileo who wrote in his 1623 text *Il Saggiatore:*

Philosophy is written in this grand book, which stands continually open before our eyes (I say the 'Universe'), but cannot be understood without first learning to comprehend the language and know the characters as it is written. It is written in mathematical language, and its characters are triangles, circles and other geometric figures, without which it is impossible to humanly understand a word; without these one is wandering in a dark labyrinth. (Translation from https://en.wikiquote.org/wiki/Galileo_Galilei)

But is "language" the right word in their meaning of "Math is a language"? We use mathematical ideas to describe the operations of the universe (or is it the other way round, the universe provides us with mathematical concepts?). But is the use of a collection of ideas the same as a language *per se*?

I do like to mull on deep things, but I am not at all good at bringing answers or conclusions to them. However, when presented with the question "Is mathematics a language?" at a moment when I have my teacher's hat on, I do have an answer, a definitive one. It is YES. And emphatic YES. And with that hat I can even tell you what language it is! For me, in the U.S., it is English. For those in France, the language is French. It is Korean in South Korea, and in parts of Afghanistan it is Dari.

COMMUNICATING MATH

Mathematical symbols are just shorthand for words. (For me, English words.) As such, any written mathematical piece is a sample writing and so deserving of all the rights and responsibilities of being a piece of writing. That is, it must be

1) a pleasure to read,
2) clear in communicating the intended information, and
3) accompanied with full and proper punctuation.

This is a shocker to many!

Think about " $x = 5$." It is a sentence. There is a noun, the unknown value x, a verb, "equals," and an object, "five." Thus, when $x = 5$ appears on the final line of a passage inked on the page, it should come replete with a period.

Question: *Find a solution to the equation* $x^3 - 100 = 25$.

Answer: From

$$x^3 - 100 = 25$$

we get

$$x^3 = 125,$$

and so

$$x = 5.$$

Notice the little connecting words, *from*, *we get*, *and so*. They are important. Notice that there is punctuation. Notice that **mathematics is human!**

It is true that I will skimp on the words and punctuation if I know I am writing to an audience already very familiar with mathematics writing:

$$x^3 - 100 = 25$$
$$x^3 = 125$$
$$x = 5.$$

(But see, I still can't resist a period at the end!) It is understood that my reader will insert the appropriate words as she reads.

Try reading the following out loud.

$$\frac{18}{8} = \frac{9}{4} = 2\frac{1}{4} = 2.25.$$

Did you naturally insert connecting words? "Eighteen over eight equals nine over four, which is two and a quarter, which equals two point two five."
(The phrase "which equals" is handy.)

Comment: I've always been an advocate of points 1 and 2, but if you look at my past writings, you'll see that I've floundered over point 3. Hopefully I am now – finally - improving my punctuation habits.

FOR STUDENTS

In the world of standardized tests graded by scantron devices and online math courses with no opportunity to write, couple with a culture of "math is about getting the right answers," the idea of putting a period after $x = 5$ seems absurd. Should we math teachers insist on good mathematical writing habits with our students?

I think we can, and should, at the very least insist on pride of presentation. We should help students work on making their mathematical output a pleasure for others to read by spacing it well on the page and by offering helpful connecting words. We should attend to helping students communicate intended ideas effectively. (This is even a Mathematical Practice Standard: *MP6: Attend to Precision*! Check it. The Common Core authors actually mean precision of language.)

Re punctuation, however, I think we should talk about it, model using it ourselves, but perhaps not insist on it. (Doing so might give a sense of tyranny.)

Some tips:

Provide plenty of blank space after questions on tests and assignments.

If we want students to take pride in presentation of their work, then give them the space to present their work! I have seen, in my tutoring, many examples of crammed quizzes and tests from teachers giving next-to-no space for students to present their efforts. I know that we're trying to save paper, but there is a cost to pedagogy. Take the ten questions stacked together in three-column format on a single page and space them two or three questions per page.

Give quizzes with all the answers supplied – and a big blank space after each question.

Make the point that the actual answers to questions are secondary (if not incidental!) Working on thinking, process, and the presentation of ideas is what it really means to be doing mathematics.

Have a classroom conversation about the use of the equal sign.

Perhaps the best way to bring attention to clear mathematical writing is to conduct a focused discussion on the use of the equal sign. In reading and writing mathematics, it is often a point of much confusion. A discussion on this one topic can do so much to steer students toward clear exposition.

Troubles usually come with strings of inequalities. The statement

$$a = b = c = d = e$$

is a (long) sentence - "a equals b, which equals c, which equals d, which happens to equal e." Such sentences often come up in mathematical writing, from writing equivalent forms of a single algebraic expression or a fraction, for instance.

Yet students tend to become "equal sign happy" and will present a sequence of algebraic steps as follows.

$$\begin{aligned} &2x + 8 = 5x - 10 \\ &= 8 = 3x - 10 \\ &= 18 = 3x \\ &= 6 = x \end{aligned}$$

This is literally saying that each expression mentioned is equal to each and every other expression mentioned. (So 6 equals 8, for instance.)

Comment: Some teachers have students use a single-lined arrow $\rightarrow$ as shorthand for "leads to" or "implies" and encourage students to write instead

$$\begin{aligned} 2x + 8 &= 5x - 10 \\ \rightarrow \quad 8 &= 3x - 10 \\ \rightarrow \quad 18 &= 3x \\ \rightarrow \quad 6 &= x. \end{aligned}$$

This is fine, except the correct shorthand for "implies" is a double-lined arrow $\Rightarrow$. This is the symbol to use from one line to the next.

Mathematicians use $\rightarrow$ as shorthand for "approaches" or "tends to." For example, in a calculus class mathematicians will write (as your students might do too)

$$x^2 \rightarrow 9 \text{ as } x \rightarrow 3.$$

for the sentence "x^2 approaches the value 9 as x approaches the value 3."

Help students understand that "$=$" is a verb and so should be used as one. Using the symbol $\Rightarrow$ for "leads to" is usually considered optional in algebraic work, a matter of personal style and taste.

Ask meta-questions about presentation.

Try giving homework and quiz questions about mathematics exposition.

Example: *A student writes on her homework*

$$4w = 20 = w = 5.$$

What does this actually say? What do you think the student was trying to say?

Example: *A student writes on his homework*

$$\frac{30r}{12} = \frac{15r}{6} = \frac{5r}{2} = 2.5 \times r.$$

What does this say? Is the student saying something reasonable?

Example: *A student writes:*

$$24y - 30 = 34 = 24y = 64 = y = \frac{64}{24} = \frac{8}{3}.$$

Can you unravel what he was trying to say?

Example: *Which is easier to parse?*

$$4(x-2y)-(3x-2(y-1)) = 4x-8y-3x+2(y-1) = 4x-8y-3x+2y-2 = x-6y-1$$

or

$$\begin{aligned} 4(x-2y)-(3x-2(y-1)) &= 4x-8y-3x+2(y-1) \\ &= 4x-8y-3x+2y-2 \\ &= x-6y-1 \end{aligned}$$

ONE FINAL IDEA:
I am hesitant to share this next idea as it must be conducted with extreme care and only when it is emotionally safe for all involved – and I am rarely confident this is actually the case. But on a couple of occasions in classes my students decided to do this next exercise.

When learning to write geometry proofs, my students decided to each publically write a proof to the same one problem on the white boards around the room. We then all sat down and just looked at the written pieces, as though they were items on display in an art gallery. Without reading the proofs we first asked "Which pieces look inviting to read? Which one looks like it might be most pleasurable to read?"

Next, after reading through all the proofs, we talked about authors' flow of ideas, their kindness to readers by not leaving too much hard thinking between steps, their style of presentation, their use of connecting words and/or actual fully worded sentences, and then, lastly, their mathematical correctness. Another group of students in one of my algebra classes decided to follow the same exercise.

Again, this activity can be extremely unsettling and disturbing to students: having one's work critiqued, even if done in the most positive of tones, is hard.

But there is something to this exercise as students in mathematics classes rarely get to see other student product and learn from it. It can be incredibly informative and powerfully instructive for our budding math writers.

HOMEWORK: Go chat with your English and Art department colleagues. What good protocols have they developed for having students critique each other's work?

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