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WEALLY COOL MATH!

CURIOUS MATHEMATICS FOR FUN AND JOY

APRIL 2015

PROMOTIONAL CORNER: *Have you an event, a workshop, a website, some materials you would like to share with the world? Let me know! If the work is about deep and joyous and real mathematical doing I would be delighted to mention it here.*

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A message from the organizers of the Algebra Readiness Challenge (ar.itemwriters.org):

ARC invites you to create a Collection of ten items using the CoreSpring Authoring Tools. Ultimately, these items will be made available so that educators across the country can use the items to help students identify their strengths and weaknesses as they work towards getting ready for algebra.

Each Collection will be scored using a trait-scoring rubric on four criteria: Aligned, Coherent, Rigorous, and Instructive. Each participant will have an opportunity to revise their Collection based on the feedback of his or her peers. Participants may also receive comments from expert reviewers and be eligible for cash rewards of up to $6,500.

Register for the Challenge today at https://ar.itemwriters.org/#register.

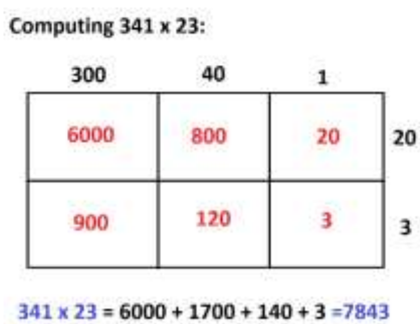
Questions? Contact info@itemwriters.org.

From me: Go for it! Create materials that are fresh, joyful, innovative, mathematically real, delightfully engaging, absorbing, interesting, and beautiful!

It is time to bring together a suite of ideas that have clearly been bubbling and brewing over the last number of essays. This month's piece is one collection of puzzles.

MULTIPLATION VIA AREA:

If ever I am required to multiply two numbers by hand (or in my head) I draw/imagine a rectangle divided into convenient pieces. After all, to me, multiplication is a geometry problem: it is the computation of area.



Question 1:
STANDARD MULTIPLICATION

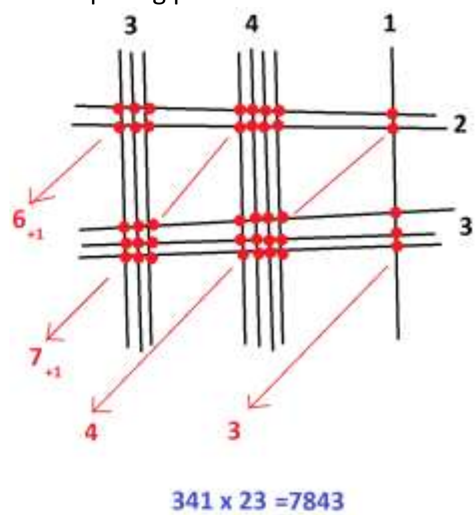
In centuries past paper and ink were precious. If one were going to use paper to do a mathematical computation, completing it with as little ink as possible was important. It is not so important today, but we still insist on teaching the "standard algorithm" nonetheless. (Why?)

$$\begin{array}{r} {}^{1}341 \\ \times \quad 23 \\ \hline 1023 \\ +\,6820 \\ \hline =7843 \end{array}$$

The standard algorithm is actually mysterious! What is it doing? Why does it work?

Question 2: LINE MULTIPLICATION

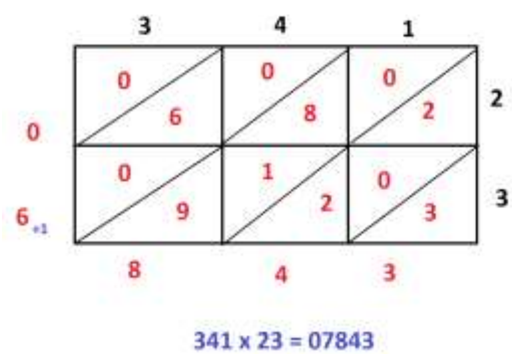
In the March 2015 CURRUCULUM ESSAY (go to www.jamestanton.com/?p=1072) I describe in detail a curious line method for computing products.



Why does this line method work?

Question 3:
ELIZABETHAN MULTIPLICATION

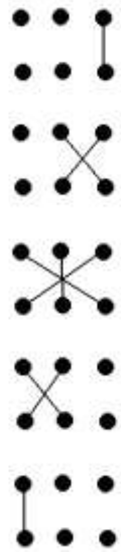
The *Elizabethan method* for long multiplication was developed in, well, England during the 1500s. (Today the method is known as *lattice multiplication* or the *galley method* for multiplication.)



What is the Elizabethan method doing and why does it work?

Question 4: VEDIC MATHEMATICS

Vedic mathematics was established in 1911 by Jagadguru Swami Bharati Krishna Tirthaji Maharaj. It has students compute the multiplication of two three-digit numbers as follows:

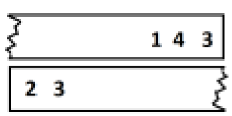


What do you think this sequence of diagrams means?

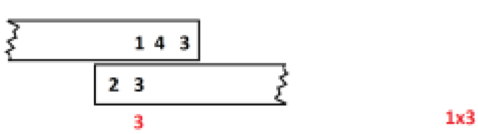
Question 5: SLIDER METHOD

Here's another way to compute 341×23.

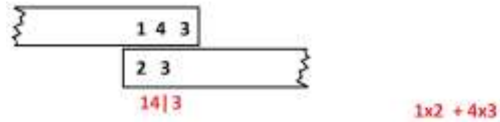
Write each number on a strip of paper, but reverse the digits of one of the numbers. Start with the reversed number on the top strip to the right of the second number.



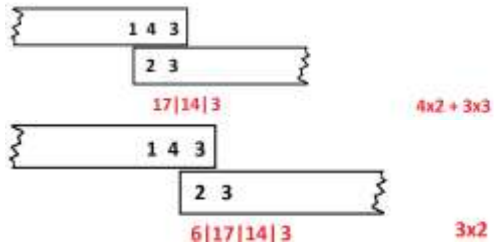
Slide the top number to the left until a first set of digits align. Multiply them and write their product underneath.



Slide one more place to the left and multiply the digits that are aligned. Write their sum of those products underneath.

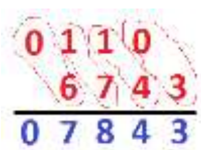


Continue this process:



The answer "six thousands, seventeen hundreds, fourteen tens, and three," that is, 7843 appears.

If you prefer, you can deal with the carries swiftly by writing individual answers as follows:



Why does this sliding method work?

Question 6: RUSSIAN PEASANT MULTIPLICATION

The following method for multiplication is believed to have originated in Russia. (Though it seems to be based on a multiplication method described by scribe Ahmes in the ancient Egyptian Rhind papyrus ca. 1650 B.C.E.)

Head two colums with the two numbers you wish to multiply.

Progressively halve the numbers in the left column, ignoring remainders, while doubling the numbers on the right. Reduce the left column to 1.

Delete all rows with left number even. Add the numbers in the right column that survive. This sum is your product.

For example, here is 37×12:

37	12
~~18~~	~~24~~
9	48
~~4~~	~~96~~
~~2~~	~~192~~
1	384
	444

Compute 23×341 this way.

Why does this method work?

Question 7: EGYPTIAN MULTIPLICATION

Here's the method Ahmes described.

Suppose you wish to compute $a \times b$.

Head two columns, the first with the number 1 and the second with the number b.

Successively double the numbers in each row. The left produces a column of the powers of two. Stop just before you reach a power of two that exceeds a.

Find which powers of two in the left column sum to a and add together the matching terms in the right column. This sum is the product $a \times b$.

For example, for 37×12 we have:

1	12
2	24
4	48
8	96
16	192
32	384

$37 \times 12 = 12 + 48 + 384 = 444$

Compute 12×37 via this method.
Compute 341×23 via this method.

Why does this method work?

GENERAL QUESTION 8:

Do any of the algorithms presented so far make it obvious that multiplication is commutative?

This next beauty is one of my favorites!

Question 9: FINGER MULTIPLICATION
Don't memorize your multiplication tables. Let your fingers do the work! If you are comfortable with multiples two, three, four, and five, then there is an easy way to compute values in the six- through ten-times tables.

First encode numbers this way:
A closed fist represents five and any finger raised on that hand adds one to that value.

Thus a hand with two fingers raised, for example, represents seven and a hand with three fingers raised represents eight.

To multiply two numbers between five and ten, do the following:

1. Encode the two numbers, one on each hand.
2. Count ten for each raised finger and remember that number.
3. Count the number of unraised *fingers in each hand and multiply together those two counts.*
4. Add the results of steps two and three. This is the desired product.

For example, "seven times eight" is represented as two raised fingers on the left hand, three on the right hand. There are five raised fingers in all, yielding the number 50 for step two. The left hand has three lowered fingers and the right, two. We compute: $3 \times 2 = 6$. Thus the desired product is $50 + 6 = 56$.

Similarly, "nine times seven" is computed as $60 + 1 \times 3 = 63$, and "nine times nine" as $80 + 1 \times 1 = 81$.

Notice that one is never required to multiply two numbers greater than five!

<u>FINGERS AND TOES</u>:
One can compute higher products using the same method with fingers and toes!

Two hands will all fingers down represents ten, as does two feet with all toes "down." To compute 17×18, say, raise seven fingers to represent ten plus seven, and raise eight toes for ten plus eight.

Since we're working with twenty fingers and toes, each raised digit is now worth 20. We currently have 15 digits up: that makes 300. Multiply the digits down, 3 and 2, to get the answer:

$17 \times 18 = 300 + 3 \times 2 = 306$!

Whoa!

Why do these methods work?

Further: Martians happen to have six fingers on each of two hands. What is the Martian version of this finger multiplication trick?

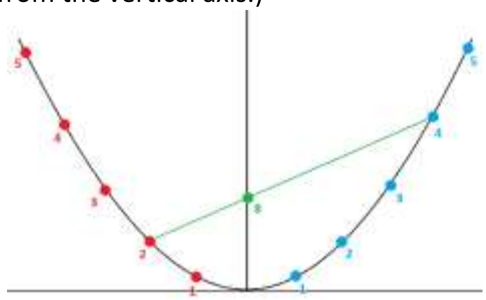
Venutians have two hands: one with four fingers and one with seven fingers. Is there a Venutian version of this finger trick?

What about Plutonians who have three hands?

Question 10:
PARABOLIC MULTIPLICATION
This multiplication method was discussed and explained in the October 2014 COOL MATH ESSAY. More about it appears in the November 2014 COOL MATH ESSAY (see www.jamestanon.com/?p=1072).

Here's a clever way to use a parabola to compute the product of two numbers.

Draw the graph of $y = x^2$ and label points along the curve by the absolute value of their x-coordinates. (So each point is labeled by its horizontal distance from the vertical axis.)



(In this picture we've just labeled the points with integer x-coordinates. For practically purposes, this is usually enough.)

Also mark the vertical y-axis with a unit scale. (**Comment:** The graph drawn here is very much not to scale!)

To compute a product of two positive real numbers a and b do the following:

Find the point to the left with red label a, the point to the right with blue label b, and lay a ruler over those two points. Read the location at which the ruler crosses the vertical axis. This location is the product $a \times b$.

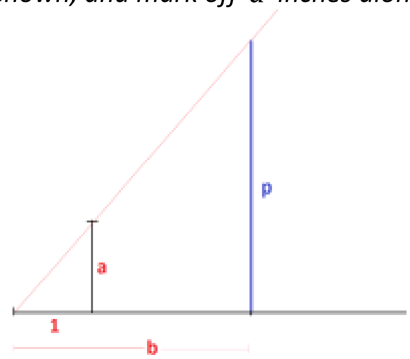
Exercise: Really do construct an accurate graph of $y = x^2$ and label the red and blue integer points on it. Use a ruler to estimate the product 2.4×3.9. Also, find an approximate value for $\sqrt{10}$.

Why does this method work? Is the parabola the only curve with this lovely multiplicative property?

Question 11: RIGHT TRIANGLE MULTIPLICATION

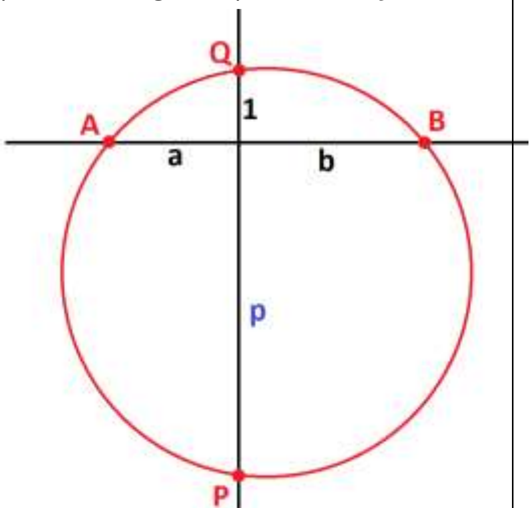
If you have a ruler and a book you can construct a product $a \times b$ of two numbers as follows:

Draw a line on a page and mark off a 1 inch length along it. Using the corner of a book, draw a perpendicular line as shown, and mark off a inches along it.



Now mark off b inches along the original base line and complete the picture shown using the ruler and the book. The length p, which you can measure with the ruler, is the product $a \times b$.

Question 12: CIRCLE CONSTRUCTION: To compute a product $a \times b$ of two numbers, draw a set of perpendicular axes and mark point A a distance a units from the left of the origin, a point Q a distance 1 unit above the origin, and a point B a distance b units to the right of the origin. Construct the circle that passes through the points A, Q, and B.



[One can construct the perpendicular bisectors of $\overline{AQ}$ and $\overline{QB}$. Their intersection gives the center of the circle. Then one can use a compass to draw the circle.]

This circle intersects the vertical axis at a second point P. The distance of the point P from the origin is the product $p = a \times b$.

Actually … The axes don't need to be perpendicular for this method to work!

COMMENT: Jonathan Crabtree at www.jonathancrabtree.com/mathematics has developed some wonderful applets that illustrate this approach (and detail some of its history). See http://tube.geogebra.org/student/m702787 and http://tube.geogebra.org/student/mk1LqCqtx .

RESEARCH CORNER

For starters, of course, one needs to figure out why each of these methods work. After that, research questions abound! For example:

1. Instead of using base two for Egyptian and Russian multiplications, are there interesting base three extensions to these algorithms? (How about a version using base three with the digits 0, 1, and -1?)

2. Do any of these algorithms naturally extend to swift methods for computing the product of three numbers? (The finger method for Plutonians, for starters? A three-dimensional volume model for multiplication and for Elizabethan multiplication? Three-way sliders and extended Vedic diagrams?)

3. Is there interesting "plane intersection geometry" worth investigating a la line multiplication?

4. What can we say about spheres intersecting coordinate axes?

Lots of fun thinking to be had!

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