



# MATHEMATICAL THINKING

## Numbers and their Algebra



*James Tanton*

(with additional tidbits by Kit Norris)

**COMMENT:** These notes are based on content from the

***THINKING MATHEMATICS!***

**Volume 1: Arithmetic = Gateway to All**

available at [www.jamestanton.com](http://www.jamestanton.com).

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## SERIES CONTENT

### EXPLORATION 1: THE POWER OF A PICTURE (18 pages)

*How long do you think it would take to add up all the numbers from one to a million and back down again? In this exploration you will learn how to do it in under one second!*

TOPICS COVERED: The counting numbers; their addition and multiplication. Clever sums and figurate numbers.

### EXPLORATION 2: FACTORS AND PRIMES (10 pages)

*Open doors and closed doors. Learn how locker doors reveal patterns in factors of numbers.*

TOPICS COVERED: Factors of numbers. Composite numbers and prime numbers. The great locker experiment!

### EXPLORATION 3: DOTS AND BOXES (24 pages)

*Exploding dots reveal the secret to understanding how numbers are written and added and multiplied and ...*

TOPICS COVERED: Place-value and its role in arithmetic algorithms.

### EXPLORATION 4: DIVISION (20 pages)

*The mysterious long-division algorithm is finally explained. It makes sense!*

TOPICS COVERED: Long division via dots and boxes. Extension to polynomial division.

### EXPLORATION 5: PILES AND HOLES (22 pages)

*How a story that isn't true can explain the mysteries of negative numbers!*

TOPICS COVERED: Basic introduction to negative numbers: addition and "subtraction," and more polynomials.

### EXPLORATION 6: THE VINCULUM AND PARENTHESES (10 pages)

*A snazzy symbol from the 1500s makes order of operations easy and obvious.*

*Why don't we use it today?*

TOPICS COVERED: Parentheses via the vinculum. Basic order of operations via the vinculum.

### EXPLORATION 7: TROUBLESOME ZERO (14 pages)

*Is zero easy to work with or tricky to work with? Is it even a number? Let's play with tricky zero and sort out all its sneaky behaviors.*

TOPICS COVERED: Is zero a number? Basic arithmetic with zero, and the danger of division. The use of zero in base 10 arithmetic.

### EXPLORATION 8: MULTIPLICATION (24 pages)

*Slicing rectangles and slicing cheese is all one needs to multiply lengthy numbers (even if those rectangles and cheeses are negatively long and wide - and high!)*

TOPICS COVERED: Expanding brackets and long-multiplication. Why the product of two negative numbers is positive.

### EXPLORATION 9: ACTING CLEVERLY (10 pages)

*Clever ways to avoid ghastly work! And clever ways to amaze your friends!*

TOPICS COVERED: Grouping and factoring. Estimation. Mathematical party tricks.

### EXPLORATION 10: FRACTIONS: ADDING AND SUBTRACTING (20 pages)

*Why do people find this topic so scary?*

*Pies and boys (and pies and girls) makes it easy!*

TOPICS COVERED: Fractions and their basic arithmetic.

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### **EXPLORATION 11: FRACTIONS: MULTIPLYING AND DIVIDING (28 pages)**

*Pies and Boys continue to save the day as we explore further properties of fractions and all the annoying jargon that goes with them.*

TOPICS COVERED: More arithmetic of fractions. Mixed numbers. Extension to Egyptian fractions.

### **EXPLORATION 12: EXPONENTS (14 pages)**

*Does folding a piece of paper in half multiple times explain everything there is to know about the powers of two?*

TOPICS COVERED: Exponents and their properties

### **EXPLORATION 13: RETURN OF THE VINCULUM (26 pages)**

*The vinculum returns to save the day in problems with division!*

TOPICS COVERED: The use of vinculum in fractions. Division in algebra. Division as multiplication by fractions.

### **EXPLORATION 14: DECIMALS (30 pages)**

*Exploding and unexploding dots return to explain decimals!*

TOPICS COVERED: Decimals and their arithmetic.

### **EXPLORATION 15: EQUALITY AND INEQUALITY (24 pages)**

*The mathematics of staying balanced - or staying unbalanced if you prefer!*

TOPICS COVERED: Properties of equality. Properties of inequality. Ranges of values.

### **EXPLORATION 16: CONFUSING THINGS THAT ARE THE SAME (18 pages)**

*Let's be honest: Many things in mathematics are confusing. Let's see if we can sort out a slew of confusing things once and for all.*

TOPICS COVERED: The word "more" and the word "less." Percentages. The word "of." Ratios. The equals sign.

### **EXPLORATION 17: NUMBERS ON THE LINE (20 pages)**

*How much of the number line actually has anything to do with numbers?*

TOPICS COVERED: Development of the number line. Rational and irrational numbers and the hierarchy of numbers. Repeating decimals. Attempts to define the reals.

### **EXPLORATION 18: SOLUTIONS**



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