EXPLORATION 8

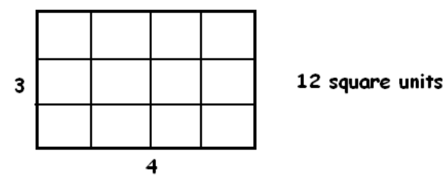
MULTIPLICATION

Slicing rectangles and slicing cheese is all one needs to multiply lengthy numbers (even if those rectangles and cheeses are negatively long and wide – and high!)

TOPICS COVERED: Expanding brackets and long-multiplication. Why the product of two negative numbers is positive.

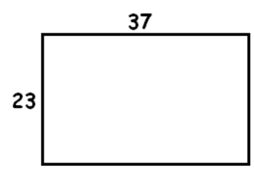
A. GETTING STARTED

We've been counting dots. Let's now count squares. Here are $3 \times 4 = 12$ squares:



This gives a picture of a 3-by-4 rectangle and subdivided into 12 squares units. (Maybe the units are inches or meters, Smoots or light-years. It doesn't matter for what we want to do in this exploration.) We like to call 3×4 the area of the rectangle. Thus we have a geometric interpretation of multiplication: The product of two numbers corresponds to the area of a rectangle.

For example, this next picture is a representation of the computation 23×37:



We can use this area interpretation to our advantage!

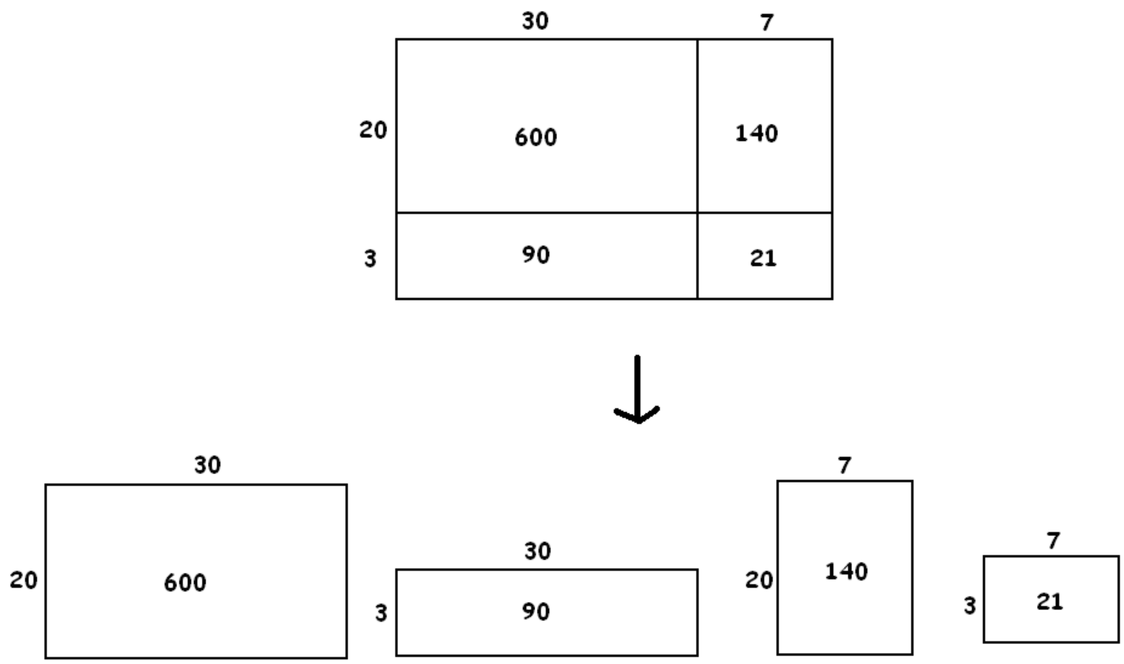
Comment: Mathematicians are somewhat coy about always regarding multiplication as area. Here we are working with positive whole numbers, and so it is easy to see that whole numbers of unit squares fit inside each rectangle. But what if the numbers aren't as nice as this? For example, does $3 \times (-4)$ correspond to the area of a rectangle? Does $\sqrt{32} \times 17\frac{3}{4}$? Maybe we should answer no, or maybe we should answer yes, or maybe we should adopt a different approach. This is what mathematicians do:

> *Let's work with positive whole numbers and see how area and multiplication should work for them. Then, we can decide whether or not then we would like to believe the same ideas should hold for all types of numbers.*

So even though $3 \times (-4)$ might not correspond to the area of an actual rectangle, maybe we can decide that numbers should work the same way as though it did!

Let's see how are can help us compute 23×37. The numbers 23 and 37, you might agree, are awkward to work with directly.

But we can simplify matters by breaking the "23" into two smaller numbers that are easier to work with, namely 20 + 3, and the "37" into 30 + 7. This corresponds to subdividing the rectangle into four pieces as shown:



The area of the entire rectangle is just the sum of the area of these four pieces. We have:

$$23\times 37 = 600+140+90+21 = 851$$

This is easy to compute! One can almost do it in one's head.

Comment: Why did we choose $23 = 20+3$ and $37 = 30+7$? (We could, instead, choose to write $23 = 16+7$ and $37 = 18+15$, say.) Because, as we saw in book 7, multiplying multiples of ten is relatively straightforward!

Notice what we did here. We wrote:

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