EXPLORATION 7

TROUBLESOME ZERO

Is zero easy to work with or tricky to work with? Is it even a number? Let's play with tricky zero and sort out all its sneaky behaviors.

TOPICS COVERED: Is zero a number? Basic arithmetic with zero, and the danger of division. The use of zero in base 10 arithmetic.

A. GETTING STARTED

Is zero a counting number? If so, what does it count?

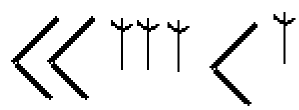
Scholars throughout the centuries did not know how to answer this question. Some societies decided, that since zero doesn't count anything, it is not a number, and so avoided it. The Babylonians, for example, of ca. 1700 B.C.E. thought this way and did not use the number zero at all. But this caused them some difficulty.

Question 1: The Babylonians were adept astronomers and based their representations of numbers on multiples of 60. (Is this because there are approximately 360 days in a year?)

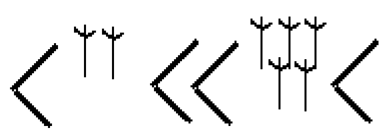
They used the symbols:

 and 

respectively, for one and ten, and use place-value notation in multiples of 60. For example,

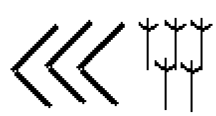


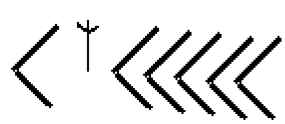
represented $23 \cdot 60 + 11 = 1391$ and



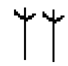
represented $12 \cdot 60 \cdot 60 + 25 \cdot 60 + 10 = 44\,710$.

a) Translate the following numbers into our decimal system:



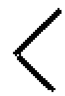


There is a problem with the Babylonian system. For example, the number 61 (which equals one 60 and one 1) is written:



which looks just like the number two!

And the number 600 (which is ten 60s) is written:



looks just like the number ten!

b) Write the number 1200 (which is twenty 60s) in the Babylonian system. Write the number 3600 in the Babylonian system. What do you notice?

Comment: Remnants of the Babylonian system still exist today. We measure time in units of 60 (sixty seconds in a minute, sixty minutes in an hour) and divide circles into 360 degrees (^{o}). Furthermore, each degree is divided into sixty minutes (') and each minute into sixty seconds (").

If, in our modern day, decided that zero wasn't a number, we too would have trouble with our place-value system for representing numbers. Recall from the $1 \leftarrow 10$ machine of exploration 3, a number such as "245" represents 2 hundreds, 4 tens and 5 units. In 2405, we have 2 thousands, four hundreds, zero tens, and 5 units. Here we counted zero tens, that is, we used zero as a counting number. If this scared us, like it did the Babylonians, then we wouldn't use it, and simply write 245, instead of 2405. (This is what the Babylonians essentially did.) But now there is a problem: 245 means a different number!

In our modern world we seem have been forced to accept zero as a number, even though it is difficult to answer what it counts!

Comment: It wasn't until the seventh-century that mathematicians began to accept zero as a valid number. Hindu scholar Brahmagupta (ca. 598-665) is credited as being the first to fully accept and describe the mathematical properties of zero.

B. THE ARITHMETIC OF ZERO

If I have five dots and add to them no dots, how many dots do I now possess? Answer: *Five dots, of course!*

This might seem a silly question, but it illustrates a key arithmetic property of zero:

$$5 + 0 = 5$$

Also, starting with no dots on a page (a blank page) and adding five dots, gives five dots on the page:

$$0 + 5 = 5$$

There is nothing special about the number five here. In general we have:

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