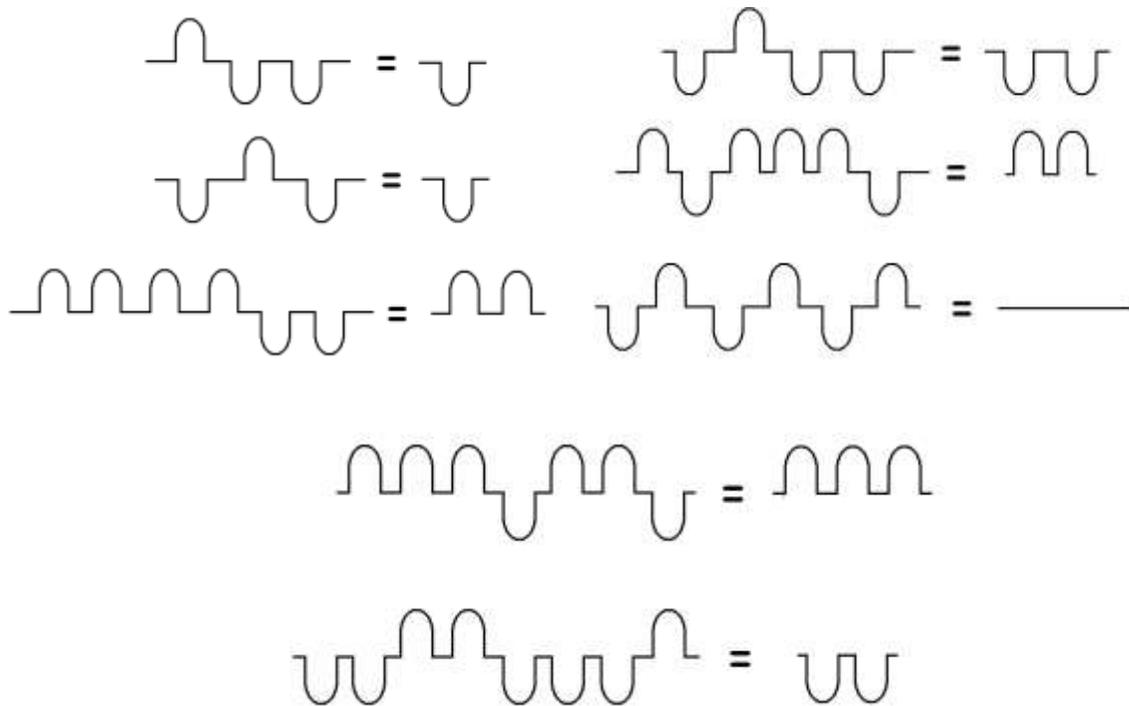


WITHOUT WORDS

Mathematical Puzzles to Confound and Delight



WW 7: SOLUTION



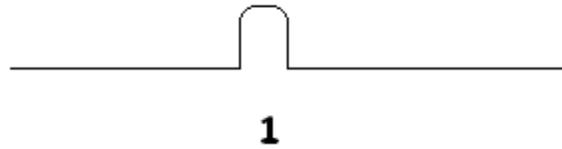
Think of piles of sand and holes in a sandbox!

Here's how I introduce positive and negative numbers in my teaching. (The following comes from chapter 4 of THINKING MATHEMATICS! Vol 1: Arithmetic = Gateway to All: www.lulu.com/shop/james-tanton/thinking-mathematics-1-arithmeticgateway-to-all/ebook/product-17511272.html.)

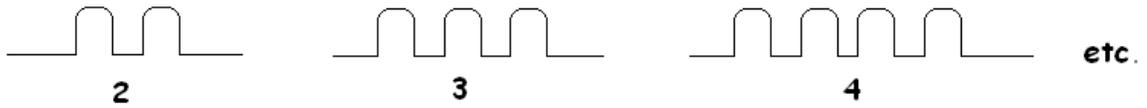
Here is a story that isn't true.

When I was a young child I would spend my days sitting in a sandbox at the back of my yard (not true). And being a very serene child I used to take my time in the morning leveling the sand in my box to make a perfectly flat horizontal surface (also not true). It very much appealed to my tranquil sensibilities, so much so that I decided to give this level state a name. I called it "zero."

I spent many an hour admiring my zero state (still not true), but then, one day, I had an epiphany! I realized I could reach behind where I was sitting, grab a handful of sand and make a pile. I called the one pile the “1” state.



And then I discovered two piles – which I called “2” – and three piles, “3” and so forth.

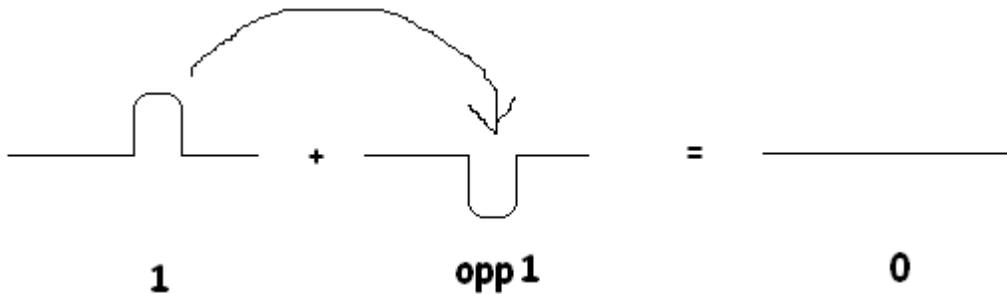


Hours of mathematical fun were had as I discovered the counting numbers with piles of sand.

BUT THEN ... one day I had the most astounding epiphany of all! Instead of using a handful of sand to make a pile, I could, I realised, take away a handful of sand and make the OPPOSITE of a pile, namely, a hole!



I called this “*opp 1*” for the opposite of one pile. And notice that “opposite,” in some sense, is really the right word because a hole “cancels” a pile:



And now you can see how this wordless puzzle – and the rest of this chapter - goes!