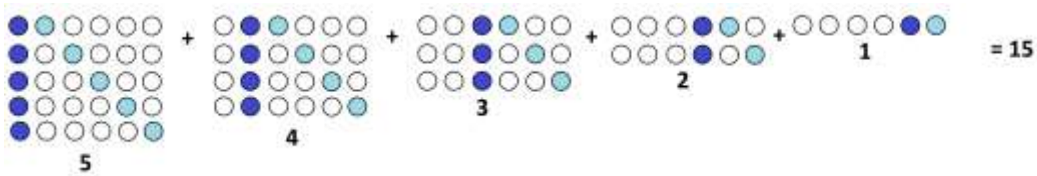
WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

WW 20: SOLUTION



If we are systematic, keeping track of the placement of the leftmost dot, we see that there $5+4+3+2+1=15$ ways to colour two dots in a row of six dots.

Similarly there are $6+5+4+3+2+1=21$ ways to colour two dots in a row of seven and $7+6+5+4+3+2+1=28$ ways to colour two in a row of eight.

In general, there are $N+(N-1)+\cdots+3+2+1$ ways to colour two dots in a row of N dots. We can get a formula for this sum.

WW 15 shows that

$$1+2+3+4+5+4+3+2+1 = 5\text{x}5 .$$

Adding 5 to each side gives:

$$1+2+3+4+5+4+3+2+1 \; {+5} = 5\text{x}5 \; {+5}$$

On the left we see two copies of the same sum:

$$\boxed{1+2+3+4 \; {+5}} + \boxed{5+4+3+2+1} = 5\text{x}5 \; {+5}$$

So $2\times(1+2+3+4+5)=5\times5+5$ and one copy is half of thisv$1+2+3+4+5=\dfrac{5\times5+5}{2}$,

which equals $\dfrac{25+5}{2}=\dfrac{30}{2}=15$, as we see at the top of this page.

The general formula is:

$$1+2+3+\cdots+N=\frac{N\times N+N}{2} .$$

This shows that there are $\dfrac{100\times100+100}{2}=5050$ ways to colour two dots in a row of 100 dots.

[See also WW22.]