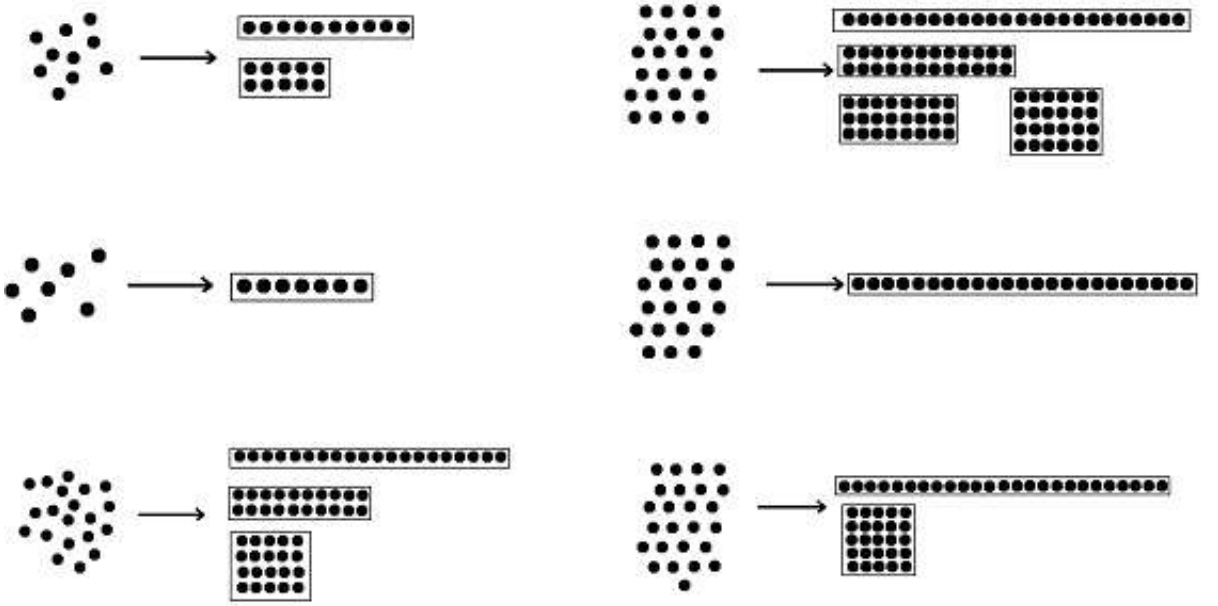
WITHOUT WORDS

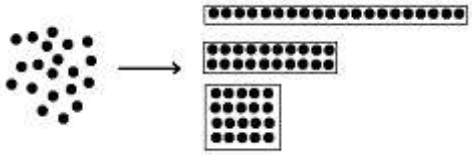
Mathematical Puzzles to Confound and Delight

WW 18: SOLUTION



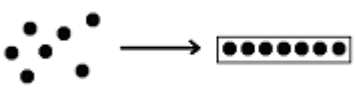
In the context of positive whole counting numbers we say that a is a *factor* of N if $N = a \times b$ for some counting number b. For example, 4 is a factor of 20, because $20 = 4 \times 5$ and 13 is a factor of 13 because $13 = 13 \times 1$.

Geometrically, the factors of a number appear as the dimensions of the rectangles one can make with that many dots. For examples, the number 20 has factors 1, 2, 4, 5, 10 and 20, and with 20 dots we can make a 1×20 rectangle, a 2×10 rectangle, and a 4×5 rectangle.



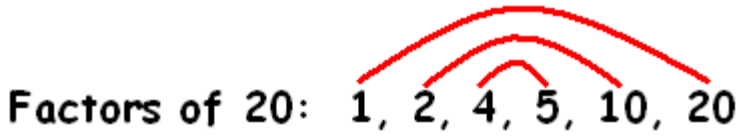
This question has us finding the factors of numbers.

Note that some numbers are resistant to having factors, and hence to making rectangles. For example, with 7 dots we can only make a 1×7 rectangle.



(Let's deem a 7×1 rectangle as equivalent to a 1×7 rectangle.) A number with precisely two factors is called a *prime* number. The number 7, for example, is prime. (And 1, with only one factor, is not prime.)

Comment: When making rectangles, it seems that the factors of a number always come in pairs:



This video is thus a surprise: www.jamestanton.com/?p=1040

For an accessible introduction to the mathematics of factors and primes see Chapter 3 of THINKING MATHEMATICS! Vol 1: Arithmetic = Gateway to All. (http://www.lulu.com/shop/james-tanton/thinking-mathematics-1-arithmeticgateway-to-all/ebook/product-17511272.html).