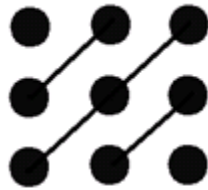


WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

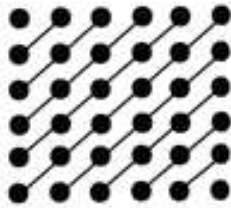


WW 15: SOLUTION



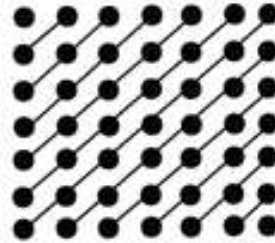
$$1+2+3+2+1$$

$$= 3 \times 3 = 9$$



$$1+2+3+4+5+6+5+4+3+2+1$$

$$= 6 \times 6 = 36$$



$$1+2+3+4+5+6+7+6+5+4+3+2+1$$

$$= 7 \times 7 = 49$$

In the same way the sum of all the numbers from 1 up to 100 and back down again must sit as the diagonals of a 100×100 square array of dots. This array has 10,000 dots and so

$$1 + 2 + 3 + \dots + 99 + 100 + 99 + \dots + 2 + 1 = 10,000.$$

The sum of all the numbers from 1 up to a million and back down again must equal $1,000,000 \times 1,000,000 = 1,000,000,000,000$, a trillion. Imagine trying to do this computation on a calculator by hand. If you punch in one number per second it would take just over 23 days just to enter the sum! But if you see the diagonals of a square in your mind, you can give the answer to the sum in less than half a second. The power of the eye and mind over the machine!

For some lovely further explorations in visualizing sums, see chapter 1 of *THINKING MATHEMATICS! Vol 1: Arithmetic = Gateway to All*. (<http://www.lulu.com/shop/james-tanton/thinking-mathematics-1-arithmeticgateway-to-all/ebook/product-17511272.html>). See also the video: www.jamestanton.com/?p=1006.