

WITHOUT WORDS

Mathematical Puzzles to Confound and Delight



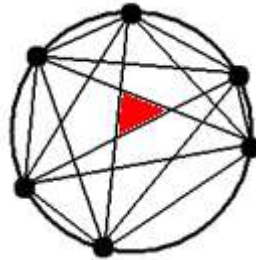
WW 10: SOLUTION



31

Surprise! Patterns need not be true!

Comment: If the dots are placed symmetrically about the circle, then three lines coincide and the center triangular region disappears to a point:



In this case one counts 30 regions for six dots about a circle. (And the doubling pattern 1, 2, 4, 8, 16, ... still fails!)

Using an asymmetrical placement of dots (so as to see the maximal number of pieces) one counts the following numbers of regions:

Number of Dots	1	2	3	4	5	6	7	8	9
Count of Regions	1	2	4	8	16	31	57	99	163

High-school students have studied this sequence of numbers and have found – and proven – a formula for the number of pieces given the number of dots. For n dots one obtains

$1 + \frac{1}{2}n(n-1) + \frac{1}{24}n(n-1)(n-2)(n-3)$ pieces. Weird! To see their work, and the wonderful

mathematics story they unveiled, have a look at chapter 4 of *MATHEMATICS GALORE! The first five years of the St. Mark's Institute of Mathematics*. (<http://www.maa.org/publications/ebooks/mathematics-galore>.)