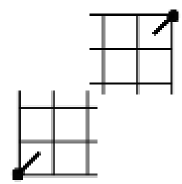
# MORE WITHOUT WORDS

***Mathematical Puzzles to Confound and Delight***

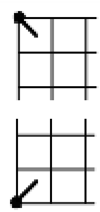
# MWW 8: SOLUTION

We're looking at the path of a bouncing ball being shot from the bottom left corner of a grid at a $45^\circ$ angle.

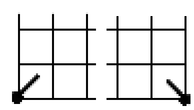
For all diagrams in the first block (which are all an odd count wide and an odd count high), the ball lands in the top right corner.



For all diagrams in the second block (which are an odd count wide and even count high), the ball lands in the top left corner.



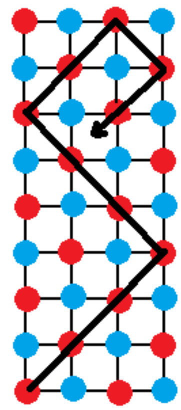
For all diagrams in the third block (which are an even count wide and an odd count high), the ball lands in the bottom right corner.



(We'll discuss the diagrams in the fourth block in a moment.)

To see why the above claims hold, color each grid point either red or blue in a checkerboard fashion, with the bottom left corner red.

We see that the ball starting on a red point forever moves on red points.



For an odd-by-even grid as the one shown, only the top-left corner is colored red and so ball can only land in the top left corner. For an odd-by-odd grid, one can see that only the top-right cell is colored red, and for an even-by-odd grid only the bottom right cell is red. Thus the corner in which the ball lands is fixed by the evenness or oddness of the sides of the grid.

> **Question:** Is it obvious that the ball must land in a corner? Could the ball ever enter into an infinite loop? Could the ball ever return to start?

The only case we have not yet considered are grids of even-by-even dimensions, as with the fourth block of diagrams.

But every even-by-even grid can be subdivided into larger "chunks" of cells and reduce to one of the odd-by-odd, odd-by-even, or even-by-odd cases.