

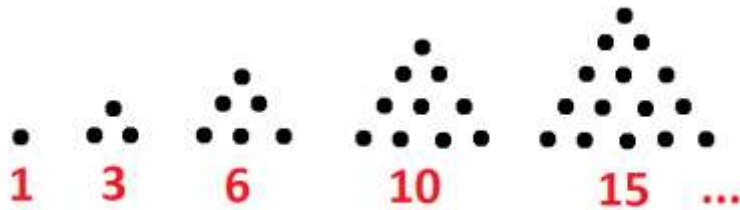
MORE WITHOUT WORDS

Mathematical Puzzles to Confound and Delight

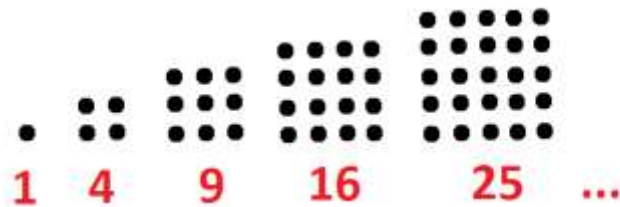


MWW 14: SOLUTION

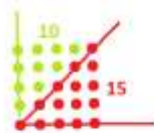
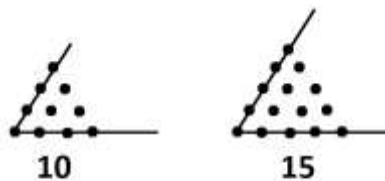
The sequence of triangle numbers begins 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, ...



The sequence of square numbers begins 1, 4, 9, 16, 25, 36, 49, 64, ...



We see from this diagram that each square number equals the sum of one triangle number and the triangle number just before it.



$$15 + 10 = 25$$

Each pentagonal number is the sum of one triangle number and two copies of the triangle number just before it: 1, 5, 12, 22, 35, 70, ...



Each hexagonal number is the sum of one triangle number and three copies of the triangle number just before it: 1, 6, 15, 28, 45, 66,



Each heptagonal number is the sum of one triangle number and four copies of the triangle number just before it: 1, 7, 18, 34, 55, 81,

Comment: We know from WW15 that the n th triangle number, which we'll denote by T_n , is given by

$$T_n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} .$$

Thus the n th square number is $T_n + T_{n-1} = \frac{n(n+1)}{2} + \frac{(n-1)n}{2} = n^2$, the n th pentagonal number is

$$T_n + 2T_{n-1} = \frac{n(n+1)}{2} + \frac{2(n-1)n}{2} = \frac{3n^2 - n}{2} , \text{ and so on.}$$