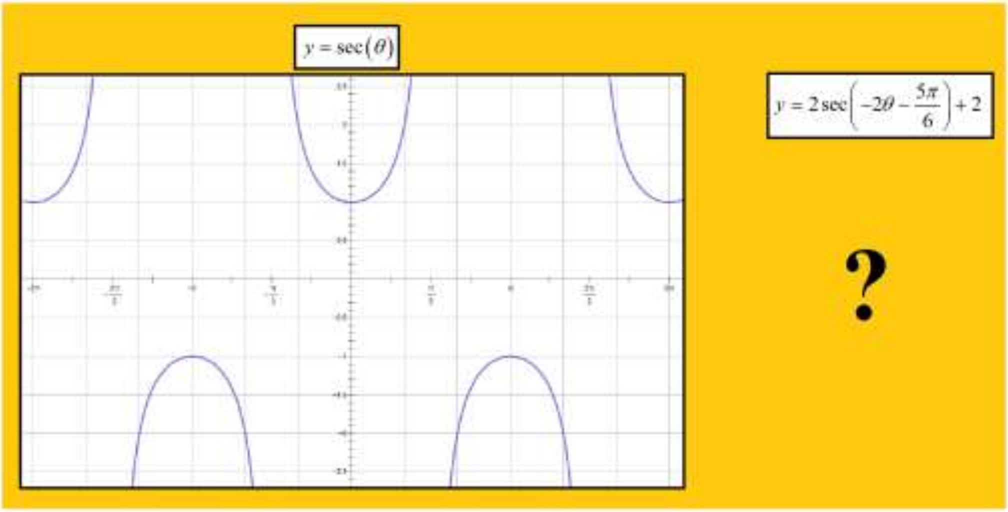

TEACHING THE PROBLEM-SOLVING MINDSET

A Classroom Moment

MATHEMATICAL ASSOCIATION OF AMERICA

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Transformations of Periodic Functions

I was recently asked to help a young lass with her mathematics homework. She was struggling sketching a graph of

$$y = 2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2 .$$

She was very comfortable with the graph of $y = \sec(\theta)$, with its asymptotes at odd multiples of $\pi / 2$ and values of 1 and -1 at the even multiples. She was just having trouble "applying the a-b-c-d rule," she said.

"What a-b-c-d rule?" I enquired.

"You know, the rule that says how to interpret a, b, c, and d in $y = af(bx + c) + d$."

My tutee was given a mental aid of some kind to help her keep straight the standard transformation-of-functions rules. She was comfortable with the effects of the first and last 2 s of the expression she was given: they double and then add two to each ouput.

(My brain thinks this way too about "a" and "d.") The trouble boiled down to the "b" and c" part, especially since the "b" is negative.

We managed to unravel the rules she was given and she successfully graphed the equation. But I couldn't resist commenting that that was hard, too hard in fact.

AVOID HARD WORK!

The graphs of trigonometric functions are somewhat repetitious. The functions are periodic after all. Surely identifying just two key points of the graph would suffice to deduce the whole graph.

For basic trigonometric functions which two inputs yield outputs that feel particularly enlightening? The inputs $\theta = 0$ and $\theta = \pi / 2$.

So consider then

$$y = 2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2 .$$

Which value of θ is "behaving like zero" for the input of the secant function?

Well, $-2\theta - \frac{5\pi}{6}$ equals zero when $\theta = -\frac{5\pi}{12}$. That is, when θ equals this value in $\sec\left(-2\theta - \frac{5\pi}{6}\right)$ we get $\sec(0) = 1$, the lowest value an upright U, and $2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2$ has value 4 and is still the lowest part of a U.

Which value of θ is "behaving like $\pi / 2$" for the input of our secant function?

We have that $-2\theta - \frac{5\pi}{6} = \frac{\pi}{2}$ when $\theta = -\frac{8\pi}{12}$. As $\sec(\theta)$ has an asymptote at $\theta = \pi / 2$, $2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2$ has an asymptote there too.

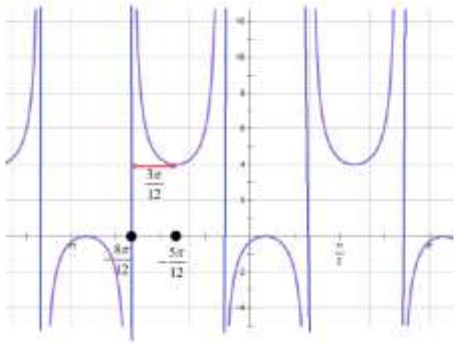
We now know the distance between a lowest point of a U and an asymptote, it's $3\pi / 12$ radians, and so the distance between neighboring asymptotes is $6\pi / 12$ radians. (Why reduce fractions if everything is being based on twelfths?)

The graph of

$$y = 2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2$$

can now be deduced.

Here's to the power of just "following your nose!"



By the way: The first step in the problem-solving process is to

ACKNOWLEDGE YOUR EMOTIONAL REACTION

I certainly first balked at the idea of graphing the beastly equation

$$y = 2\sec\left(-2\theta - \frac{5\pi}{6}\right) + 2.$$

My personal reaction to being asked to graph this was: "Why?"