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TANTON'S TAKE ON …

EXCLUSIONARY MATH

MARCH 2017

I run the risk of this being a bit of a snarky essay. The reason is that I am not really sure whether or not I mean it to be!

My trouble is that I am deeply saddened by many of the common practices inserted in the school curriculum and forced upon good thinking teachers and good thinking students. Many folk feel powerless to "cut through the clutter" and simply encourage the use of common sense to nut through challenges. Our culture is set on insisting that mathematics is hard and that its study needs to be thoroughly scripted and broken down into carefully orchestrated, if not overly orchestrated, steps.

A basic consequence of this is the demand to constantly "explain your reasoning." Two disconcerting examples came my way just last week.

A parent of a youngster asked me how to get her son to explain his work on quizzes and tests. He's receiving failing grades because he is not explaining how he knows that 120 divided by 6 is 20, for instance. Poor lad! Why would he want to explain something so obvious if he can just see it as so?

But I also completely understand the teacher's predicament. I do not doubt that he or she realizes that this lad "gets it." A teacher's instinct is to then ask such a fellow a more demanding question, one that's got some thinking worth explaining in the lad's opinion. "What do you think 120.3 divided by 6 might be?" might get him writing. If he does he will no doubt demonstrate along the way how he thinks of 120 divided by 6. But there is no time for such individual attention and questions that aren't aligned with the standards for the unit are sometimes seen as *verboten* by school supervisors.

Next I was asked to help a friend's daughter with an upcoming test on graphing rational functions. She was given a series of twelve steps to follow, the final one being – finally! - to sketch the graph of the function. She was to be graded on each of the twelve steps. (This is the high-school equivalent of "explain your reasoning.") She gave me an example from the practice test, which was something like.

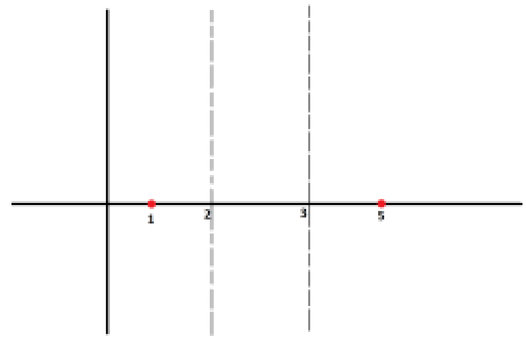
Sketch the graph of $y=\dfrac{2(x-1)(x-5)}{(x-2)(x-3)}$.

My first piece of advice was: no one says you have to answer a set of questions (or do a sequence of steps) in the order presented to you. I showed her my approach to such a problem. I call it the "flounder and just follow your nose" approach. Here how it goes for me.

Are there any interesting x*-values staring us in the face?*

Well, yes: the values $x=1$, $x=5$, and $x=2$, $x=3$ seem interesting.

Each of the first two values produce $y=0$ giving us two points we can plot on the graph right away. Wooho! The latter two x-values are "dangerous" as they each give division by zero: we can't go there! Let's just cross out each of those dangerous x-values with a dotted line.



Personally, I think there are two more interesting x*-values: an extremely large positive one, like* $x=10,000,000$*, and an extremely large negative one, like* $x=-10,000,000$.

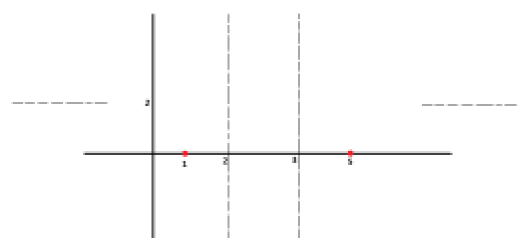
For $x=10,000,000$ we get that y is really close to

$$\frac{2\times 10,000,000\times 10,000,000}{10,000,000\times 10,000,000}=2,$$

and for $x=-10,000,000$, y is really close to

$$\frac{2\times(-10,000,000)\times(-10,000,000)}{(-10,000,000)\times(-10,000,000)}=2.$$

Let's add this information to the picture.

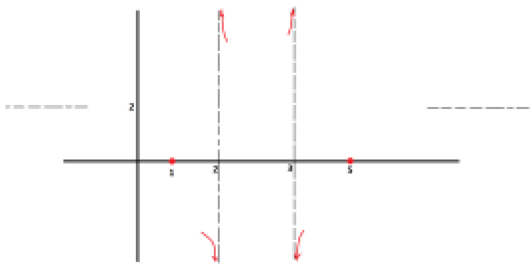


What about those danger numbers? What if we are quite daring and walk right up close to them? (Who doesn't like the edge of danger?)

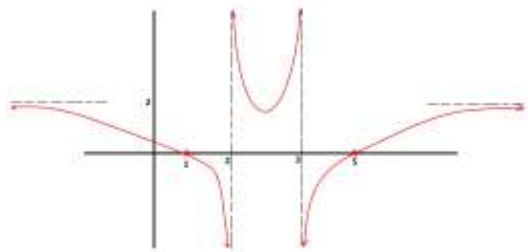
Just to the left of $x=2$, say at $x=1.99$, we get

$$y=\frac{2\times(0.99)\times(-3.01)}{(-0.01)\times(-1.01)}\approx\frac{2\times 1\times(-3)}{-\text{small}\times(-1)}$$
$$=\text{large negative}$$

Similar work for just to the right of $x=2$, and just to the left and right of $x=3$ gives the following picture.



Since the graph cannot touch the x-axis again, its picture must be something like this.



The domain and the y-intercept of the function are now incidental details. (Finding the range of this particular example, however, is not easy! Is $y = 2$ part of the range? Is the middle section of my graph reasonable as drawn?)

Sketch the graph of $y = \dfrac{x(x-1)^2}{4(x-4)(x+4)}$ **.**

This example has an oblique asymptote, and my tutee was able to handle it. (We have six interesting x-values: the four "obvious" ones, along with $x = \pm 10,000,000$. Do you see these latter two make you think of the diagonal line $y = \dfrac{1}{4}x$?)

This next question is so straightforward that it is hard!

Sketch a graph of $y = \dfrac{x-6}{x-6}$ **.**

As a high-school teacher I always strove to teach kids to "follow their noses" rather than follow a prescriptive procedure. After all, if I really wanted them to get the graph of $y = \dfrac{2(x-1)(x-5)}{(x-2)(x-3)}$ I'd advise them to pull out their smart phones and use *Wolfram Alpha*. (We are in the 21st century after all! We're no longer teaching to get answers.)

So why was this lass given a twelve-step procedure to follow? And why was she to be graded on all twelve steps?

The president of a prestigious university once said to me that high-school mathematics is taught to exclude. I am not sure if I agree. At least, I do not believe the intention is to exclude. But I do wonder if it is, in the end, the net effect? Yes, we provide a twelve-step procedure to help everyone through. But look at the context: a twelve-step procedure to answer one type of question no one particularly cares about in the first place. This can well appear as a deliberate ploy to be arcane and off putting.

Society puts mathematics on a different pedestal than other subjects. Do we equate "intelligence" with ability in any subject as readily as we do with mathematics? If you are good at literature, are you immediately labeled as smart? If you are good at math, you must be smart. Is there such a thing as a history genius?

Math is portrayed as hard. I am not sure which came first: the perception that math is hard or the need to produce such an overly scripted curriculum that gives educators little freedom to have human conversations with their students, the freedom to teach life skills such as "following your nose." Every curriculum topic must be pre-broken into accessible and manageable steps for students to follow, thereby reinforcing the perception that math must be inaccessible and hard.

I recently gave a big public talk on the joy of mathematics at a highly-recognized community college, a talk open to the entire college community and the general public. But not a single college administrator was available to attend the event – all had prior commitments. The college mathematics department was disappointed. I wasn't surprised. The societal message is that math is arcane and hard comes across loud and clear. Intelligent administrative folk, like all folk, are very good at getting the message!

Math is inherently human, inherently accessible, and inherently joyful. I dream of a curriculum that does less, much less (but I will settle for one that, at least, prescribes so much less). Of course students might need the help of a twelve-step procedure every-now-and-then, and that is fine, as long as *they* were part of the conversation that devised the procedure! Let's make mathematics, even the arcane mathematics, inclusionary rather than, by sad mis-intent, surreptitiously exclusionary.

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