CURRICULUM INSPIRATIONS: www.maa.org/ci

MATH FOR AMERICA_DC: www.mathforamerica.org/dc

INNOVATIVE CURRICULUM ONLINE EXPERIENCES: www.gdaymath.com
TANTON TIDBITS: www.jamestanton.com

TANTON'S TAKE ON …

THE RENEWED MATH WARS

JANUARY 2016

The following is an essay I recently posted on Facebook. Some passages appear in my previous writings and I've added some additional thoughts since that first posting.

QUESTIONS FOR THE NEW MATH WARS

There's a call across the U.S. in response to the Common Core State Standards, and a call in parts of Canada too with their curriculum reforms, to GO BACK TO BASICS, to take mathematics learning for our kids back to what it should be, the mathematics we know and the mathematics we recognize. The call is vehement, and in some communities, surprisingly heated. But it is always founded on genuine concerns parents have.

I found myself caught in this controversy when I was recently asked to give a mathematics lecture for parents, teachers, administrators, and students. (View the talk here.) One phrase used to advertise the event seemed to serve as the trigger for social-media and public media pre-talk outrage: "Experience how the power of understanding trumps memorization."

Why outrage?

The word "trumps," it seems, was the trigger. It unwittingly thrust me into the center of a binary argument.

The debate about the appropriate nature of mathematics learning and doing for our children has turned to the levels of the math wars from decades ago. Is it a war about understanding *versus* memorization? What is it about, really?

I certainly have no answers. But I do have some fundamental questions that I feel would ground matters and bring us to good conversation, if they were explicitly answered. Without such grounding, I fear this debate is amorphous and dangerously reactionary, and therefore anything but productive.

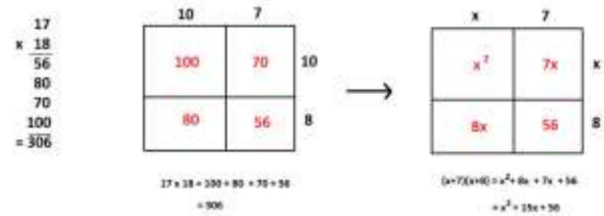
SIX QUESTIONS ABOUT THE MATH WARS I WISH WERE ANSWERED

1. What is the context of the conversation? What grade levels are we talking about?

It seems to me that the majority of debate revolves around having students memorize the multiplication tables. Am I right about this?

Parents, all adults in fact, should absolutely be cognizant and invested in out next generations' education, all the way through, from K though 12. The conversations I am seeing and hearing are primarily about the early grades. Is there equal concern and discussion about the higher grades?

My fear is that over the past century we have a developed a society generally afraid of mathematics (with accompanied societal pride to publically not like mathematics). Consequently, parental attention to education is focused on K-7 mathematics. There is too much fear to comment on, or even look at, the state of mathematics education in grades 8 – 12. Is the math-wars debate consequently terribly unbalanced? Are all "calls for action" fully mindful of the full K-12 storyline our students will be experiencing? The phrase "understanding trumps memorization" actually comes from my work in high-school mathematics, when discussing polynomial division and the like. At that level, I really do feel that understanding the mathematics of polynomials is far more valuable and productive than memorizing the mechanics of their algebra. (Does this context for the phrase still incite outrage?)



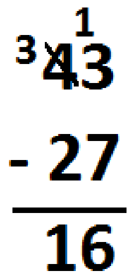
Connecting grade-school arithmetic with geometry and with high-school algebra: seeing a single story throughout mathematics.

Youngsters should, of course, at the age-appropriate point, know their multiplication facts so that they are not held back stumbling over small details as they move forward. And as many back-to-basics proponents point out there is research to show that memorization tasks do help with developing some types of good cognitive function. Another question of context: Is that research also suggesting that a focus on memorization is appropriate for all levels of mathematics teaching – middle school, junior high, senior high? Is the evidence suggesting this the basis of best practice for mathematics learning in all grades, all the way through? Of course, the art of memorization is appropriate for students throughout many subjects in school. Must a focus on memorization be conducted in math class because that cognitive development doesn't happen well enough in other areas of schooling? What is the context for the power-of-memorization argument?

2. What is the difference between familiarity and understanding?

I don't mean this to be a snarky question, but it is very easy for us all, each a human being, to equate familiarity with understanding (and non-familiarity with dismay!)

Consider, for example, the subtraction problem 43 take away 27. Those of us trained in the traditional ways would expect to see this problem worked out this way:



This is so very familiar. We adults feel we fully understand what is going on here because we recognize what we see and can do this same approach ourselves swiftly, without even thinking about it. We just do it.

But do we understand it? Dare I ask questions about the algorithm?

Why do we start from right to left? (After all, we are taught to read left to right in school. Why this switch of direction in math?) When starting from the right, why can't I write three take away seven is negative four? That is actually correct. Why do we prefer to "borrow a one"? And when we do, is it actually valid to magically change three to thirteen? (At face value that seems mighty strange.)

Perhaps these questions are just irritating and it is better to not ask them – just do the algorithm to get the answer. But why then do the algorithm at all? In this day and age the best and most reliable away to get the answer to an arithmetic problem is to pull out one's smartphone. I believe youngsters tend to have smartphones too these days.

But we almost universally object to the idea of youngsters solving arithmetic problems with calculators as there seems to be negligible thinking and understanding taking place with calculator work.

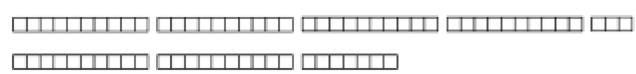
So where does this leave us in teaching subtracting to third- and fourth-graders? Teach the algorithm, but not for understanding, just for pen-and-paper doing, even though this is not the way to perform the computation in this day and age. Maybe we can argue that practicing the algorithm encourages fluency with math facts? (If so, is this the best way to promote such fluency?) We adults really need to be clear and honest about what we require our children to do and work through. The answer cannot simply be "they must do this because it is familiar to us." We need to articulate sound, considered reasons.

Now comes a friction.

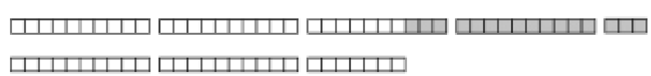
What if your child solves 43 take away 27 as 3 + 10 + 3 = 16? Your child just did something. Your child got the correct answer. But whatever your child did is certainly unfamiliar.

Have we adults, perhaps having only been taught the traditional algorithm for subtraction, been given the skill of flexibility to figure out what is afoot here? My fear is that the answer is, by and large, sadly no.

Maybe your child is seeing the subtraction visually, as two rulers side-by-side, one 43 units long and the other 27 units long.



In which case, the difference in their lengths naturally breaks into three sections: one 3 units long, one 10 units long, and a final one 3 units long. The difference is 3 + 10 + 3, that is, 16 units.

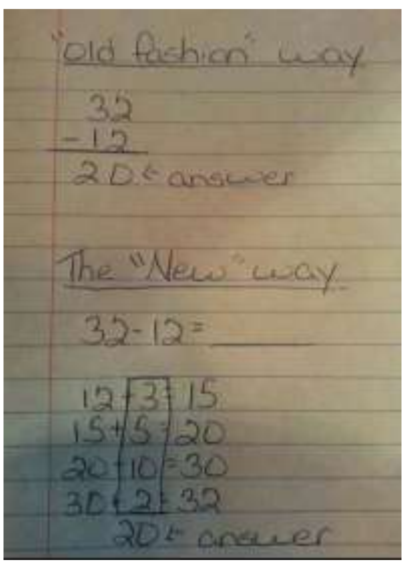


Maybe to your child thinks that writing out the traditional algorithm is a cumbersome way to solve this particular subtraction problem given that she can just see the answer in her head! (What's 203 – 186? Can you now *see* the answer to that in your head?)

It is true that this example assumes familiarity with some kinds of basic "math facts:" that two consecutive multiples of ten differ by ten, that 27 is three below a multiple of ten, and so on. The context of this example is thus the appropriate grade-level for which the student has sufficient fluency with these features of arithmetic. (Or should I say "memorized"?)

3. Can internet examples be believed?

The exemplar of absurdity making the internet rounds is this example, showing how to compute 32 – 12 the "old fashioned way" and how to compute it the "new way." One can only look at this page and say egad!



I personally can't help but suspect that the author of this script deliberately chose an absurdly straightforward subtraction problem to illustrate a point. The answer to 32 take away 12 is 20, after all. (Did you even need the "old way" to see that?) And if this is a true example from a student's homework packet, then, I agree, requiring a student to write out absurdly detailed steps would be, well, absurd.

But I sincerely worry about the lack of support of teachers caught in the middle of these math wars experience. They are parents and people too. And when a change in the curriculum comes along, they may well need help and support in making sense of it. The trouble is that they are often put in the spotlight right off the bat and their work, as they try to figure things out, is held up as exemplars of inanity and badness. I hope we can always be understanding and kind.

We are all, of course, on board in wanting to teach kids nimble, flexible thinking, and the confidence to solve problems in both standard and new contexts. To this end a new curriculum might suggest that teachers might explore a range of strategies with their students for solving a subtraction problem, say. Do 205 – 168 with the standard algorithm, or be clever and save yourself some work and subtract five from both numbers and do the problem 200 - 163 instead and just see the answer in your head, or if you don't quite trust your head yet, subtract one more from each number and make it 199 – 162 and do the standard algorithm but now cleverly avoid all the tricky carries. (A student reasoning this way is demonstrating exceptional mastery of the long subtraction algorithm! Wouldn't it be lovely to strive for such mastery for all students?)

However… If a new teacher interprets the idea of multiple strategies as an *edict* to teach all the strategies and test students on all the strategies (which usually requires inventing names for the strategies, to be memorized), and mark students wrong if they solve a question correctly but with the wrong strategy, then we have a problem. A serious problem!

Is this the issue serving as the basis of concern for the call to GO BACK TO THE BASICS? The internet examples I see as "proof of absurdity" tend to be of this universally agreed inappropriate extreme.

If a teacher or an administrator dictating classroom practice is having trouble understanding context, then I say we should guide and support that teacher! We are all striving to help each and every student develop his or her own mastery and flexibility with the mechanical work of mathematics (and more). A set of options, if taken as edicts to all be enforced, can only serve to confuse and demoralize. No one wants that for our students!

4. Do we trust the teachers of our children? Do we see teachers as expert professionals?

Putting exceptional examples aside, do we, as a community of parents, trust the expertise of teachers? Do we value the teaching profession as a profession?

The answer to this questions is surely mixed – at least for the teaching at the grade levels whose mathematics content we feel we can comment on. In the U.S., at least, it is not even clear to me that the teaching profession itself always values its teachers as experts and professionals. The administrative system of day-to-day accountability that mathematics teachers often face is so restrictive that there is next-to-no flexibility in the classroom. Also, curriculum is often over-scripted and teachers, it seems, can really be no more than transmitters of that curriculum, certainly not experts conversing with their students about a curriculum. (It is curious that the Common Core English Language Arts Standards are much more openly worded than the Common Core Mathematics Standards, for example.)

As such, much of the parental mathematics debate in the U.S. is about the content being covered, which textbook is being used, and rarely, if at all, about the implementation of the content. If mathematics is only seen as an edifice of facts and computational techniques to be communicated to and then mastered by students, then, naturally, "memorization" feels like it should, in some vague way, play a prominent pedagogical role. (In what way exactly?) In this worldview, mathematics is limited to answering "what" questions. But nowadays international tests not only test math facts questions, but also thinking and problem-solving questions. There are "what else?" and "why?" questions to attend to too. (And to foster innovative and deep-level problem solving, there are "what if?" as well.)

If we, as parents, don't immediately see certain grammar of mathematics being drilled at a particular grade-level, why is it so easy for us to presume the grammar isn't being attended to at all? Who or what is it we don't trust?

5. Is the internet jargon defined?

In my readings of materials calling for a return to basics in math education, I see hot-button terms such as "discovery learning" and "inquiry based curriculum" bandied about as though they are in standard use in education consortia and in a State's or Province's curriculum. Here's the thing: all curriculum standards are available in full online and it takes next-to-no effort to search a document for these terms. It is important, and appropriate, to see the printed definitions of these phrases as allegedly used by these new reform curriculums: the names are too vague to have stand-alone meaning. Concerned parents can easily double-check the basic grounds of any reform or anti-reform argument and check any claims made. We are all striving to be informed citizens of the world. We should each take that same level of responsibility here too. (By the way, I am yet to find a curriculum document using the phrase "discovery learning." I personally cannot begin to guess what this phrase actually means.)

6. What is the balance between understanding and memorization? Memorization and fluency?

The UK is moving to require all students to have memorized their times tables up to twelve-times-twelve by age eight. Helping students do so is seen as an educational entitlement, as not knowing such basic mathematics facts, it is claimed, causes anxiety. Parents, if they believe this is right for their children here in North America, can always choose to help their students this way too, irrespective of what is going in the school classroom. (Has anyone made that point?)

So is memorizing the quadratic formula the right thing to do too? Is memorizing the rules of trigonometry vital too? The proposition numbers of Euclid's geometry theorems? All the log rules? Again, are we talking only computation and arithmetic facts?

As a professional mathematician I've never memorized the quadratic formula. I can derive it if you want me to. But in solving a quadratic, using the formula is usually an unenlightening way to work through the challenge: playing with my understanding of symmetry often leads to new insights and advances in thinking. The fact that I can recite the sine of 45 degrees is only because I've had to work with the value of this quantity multiple times, and now it is in my just my head. I will never memorize the log rules as they all just follow logically from the basic definition of logs. Once I've got that understanding, there is nothing for me to store in my head.

As a college professor I wanted – needed - students who could "nut their way" through challenges. Memorized formulas only serve to solve problems for which those formulas are apt. That's not real-world mathematics. Math, like life, is full of organic, messy, challenges, with no answers in the back of a book.

So then, where is the balance between memorization and understanding? The K-12 curriculum has to think very hard about this balance and there is no universally correct answer. (Again, we need to make sure that whatever answer we settle on is fully mindful of the whole K-12 story.)

WHERE DOES THIS LEAVE US?

It seems to me that all parties are philosophically on the same page. We are all deeply, and rightly, concerned for the future well-being of kids and of our next generation as a whole. We want a next society of confident and competent problem solvers, ones who can take on the big challenges of the globe and make significant headway with them. And mathematics, of all subjects, seems to be seen as the key for teaching confident and effective problem-solving. We adults are therefore rightly passionate about the state of mathematics education for our children, and we have been for decades at the very least. "New Math" was developed in the late 50s. A call to go "back to the basics" occurred in the 70s. And here we are again. Since mathematics is seen as vital towards our nation's future success, it will always be a highly political topic – and a highly emotional one.

So. One final question: Where do we go from here?

tanton.math@gmail.com