



CURRICULUM INSPIRATIONS: www.maa.org/ci

Uplifting Mathematics for All

www.theglobalmathproject.org



INNOVATIVE CURRICULUM ONLINE EXPERIENCES: www.gdaymath.com

TANTON TIDBITS: www.jamestanton.com



TANTON'S TAKE ON ...

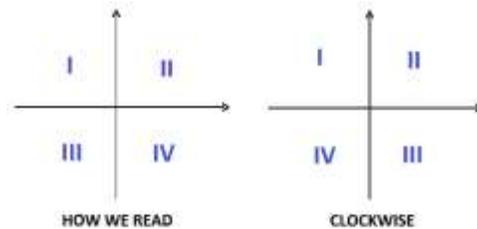


COUNTERCLOCKWISE

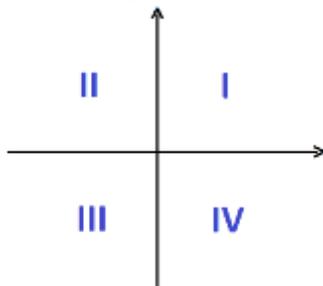


FEBRUARY 2017

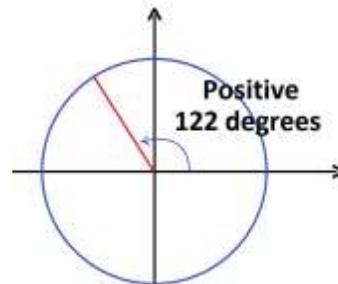
Why is counter clockwise the preferred direction of rotation in mathematics when the rest of the world naturally thinks clockwise?



Our labeling of the four quadrants of the pane is quite strange.



Why do we measure angles from the horizontal line to the right in a counterclockwise direction? Why this mismatch with everyday life?



Why not label them as we read, left to right, or at least label them in a clockwise direction?



TIME

The preference for counterclockwise in mathematics isn't a mismatch with society. It is actually a logical consequence of society's preference for associating clockwise motion with the passage of time.

In the northern hemisphere the Sun hangs in the southern sky. So it is natural to face south when examining the motion of the Sun. And in doing so we see that each day the Sun rises from the left, follows a large arc, and sets to the right. That is, the Sun follows a particular direction of the rotation, which became known a "sunwise."

The old English word for sunwise is *deasil*, which, according to Merriam-Webster, is based on the Scottish Gealic *deiseil*, from old Irish *dess* (meaning southern or right) plus *sel* (for turn). This is believed to be related to the Latin word *dexter* for right handedness. (The Sun moves to the right.)

The sunwise direction of rotation was considered good and lucky. Many rituals have participants circle or turn in a clockwise rotation. Meanwhile, the reverse direction, *widdershins*, is often associated with evil. (I believe it is standard in the play *Macbeth* to have the three hags circle their cauldron widdershins.)

The sunwise direction also matches the motion of the shadow of a sundial.



So with the invention of time pieces it was natural to set the rotation of hands to

match the sunwise rotation. That is, sunwise became clockwise.

Question: *I grew up in Australia. What might be the societal convention if Aussies had their way?*

Clocks have the hour of 12 placed at top of their faces. Is that because the Sun, at noon, is up at its highest point?



MAPS

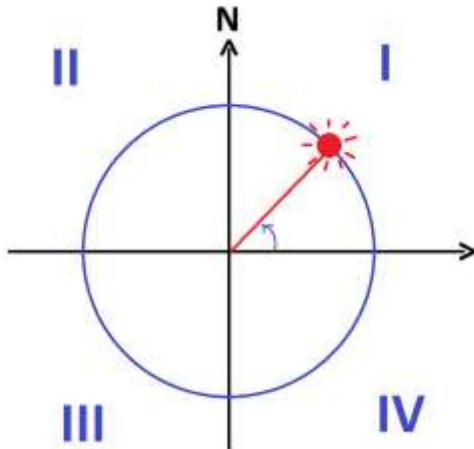
All through this, scholars and explorers were drawing maps. It is appropriate to indicate the four directions North, South, East, and West on a map. But which direction should one assign upwards on the page?

Ancient Egyptians drew maps with south facing upwards. (Is this because, in holding up the map, the Nile then flows downward, thus matching the force of gravity?) Many ancient Chinese and Middle Eastern maps have south depicted upwards too. This idea also appeals to modern-day Australians. A large number of maps from medieval Europe put east upwards.

Greek scholar Ptolemy (ca 100 AD) conducted a significant amount of work in cartography. For unclear reasons, he often set north upwards on his maps. During the European Renaissance, with the blossoming of world exploration and renewed interest in cartography, scholars of the time referred to Ptolemy's work and seemed to have simply followed suit in setting north up. And the tradition has now stuck.

Well one cannot help speculate the consequences of these societal choices. Did holding maps upwards naturally associate north with the upward direction? We talk about "going up north" or "heading down south."

Next came the Cartesian plane in mathematics. It has a direction which we might think of as up, and hence as north.



Linking “up” with north then naturally sets east to the right. (Think of where east is when you are facing north.) The remaining two directions are also set: south is down, and west is to the left.

So much of mathematics is motivated by the study of astronomy: the motion of the stars, the Moon, and the Sun. Trigonometry, it seems, is the study of the height and the displacement of the Sun at different observed angles of elevation.

Now look at our model of the Cartesian plane with “up” linked with north. In this model the Sun rises from the eastern horizon (the right horizontal axis), moves upward through the top right quadrant (this is the first quadrant of the plane it traverses), reaches a pinnacle and then moves through a second quadrant of the plane, to then set in the west. During the night, the Sun then passes through a third and then a fourth quadrant.

We are left with a model that naturally measures angles counterclockwise from the right horizontal axis and with the quadrants of the plane numbered in a curious counterclockwise fashion.

Question: *How might our Cartesian coordinates conventions change if cartographers decided, in the end, to set south or east as “up”?*

~~~~~  
**RESEARCH**

Mathematics is a wonderfully human story, full of curious choices, conventions, and ramifications. But it is often hard to get the historical story straight!

I’ve peppered this essay with phrases like “it seems” and “one cannot help but speculate,” as I cannot find clarity on the historical issues I presented here through my own personal research, reading, and discussions with historians.

Did mathematicians in the 1600s overtly link the Cartesian plane with cartography? Is a link between “north” and “up” explicitly outlined?

Feel free to delve deeply into these questions and related issues to set me straight on my speculations!

~~~~~  
© 2017 James Tanton
tanton.math@gmail.com