

★ **WAAH!! COOL MATH!** ★

CURIOUS MATHEMATICS FOR FUN AND JOY



JUNE 2013

PROMOTIONAL CORNER:

Have you an event, a workshop, a website, some materials you would like to share with the world? Let me know! If the work is about deep and joyous and real mathematical doing I would be delighted to mention it here.

My first of many new online experiences for educators and students is up!

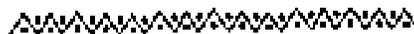
QUADRATICS: *How to understand them (and how to teach them) with joy, and ease, and not one whit of memorization!*

CHECK OUT: www.gdaymath.com

(The site name is really "Hello Kitty" except it is an Australian hello. And instead of greeting a kitty we are greeting math!)

Registration for the NY Math Circle Summer Workshop for middle-school educators is now open!

VOLUME and SURFACE AREA
July 29- August 2, Bard College
www.nymathcircle.org/workshop

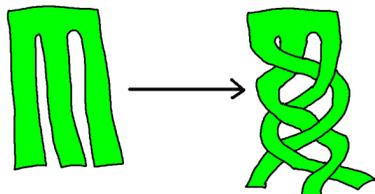


PUZZLER: Pick up a piece of string from a table top, one end of the string with your left hand and the other end with your right hand.

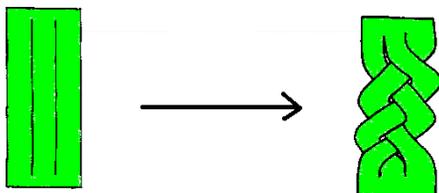
Now, without ever letting go of either end of the string, maneuver your arms and your body so as to eventually tie a knot in the string. It can be done!

NOT-SO-IMPOSSIBLE BRAIDS?

To make a braid one usually starts with three parallel strands joined together at one end but kept loose at the other.



But why bother with the loose ends? Go ahead and make a braid with no free ends! It can be done!



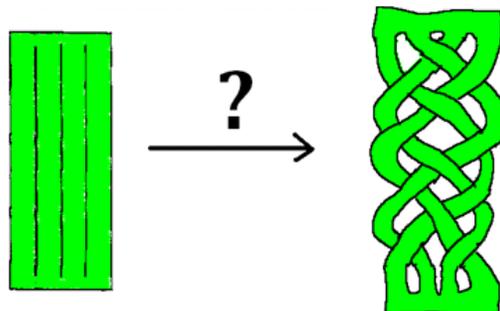
Here's me holding a no-free-end braid made from paper. Notice that the individual strands are relatively flat: there are no twists and the same one side of the paper faces outwards at all times within each strand.



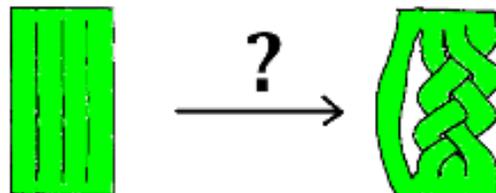
Comment: Paper is very hard to work with – it is annoyingly inflexible! I recommend cutting two slits in a rectangle of felt and playing with that instead.

CHALLENGE:

Why stop at three strands? Can you make this no-free-end version of a four-strand braid? Try it!

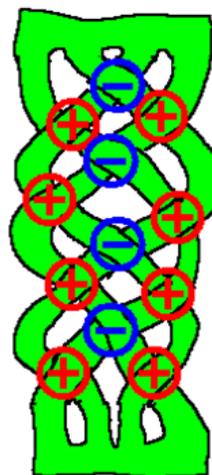


How about a three-braid with an extra strand along for the ride?



GIVING SOME THINGS AWAY:

Let's look at the four-strand braid. Notice that strands cross a total of 12 times, sometimes with the right strand crossing over the left (let's call these positive crossings) and sometimes with the left strand over the right (negative crossings).

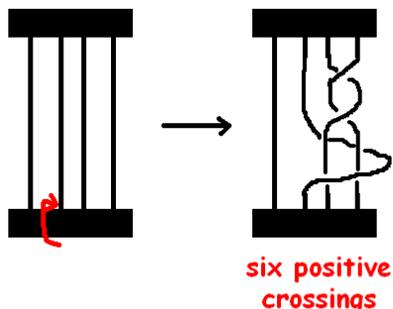


Index = +4

The diagram we hope to construct has four more positive crossings than negative crossings. Let's say it has index +4.

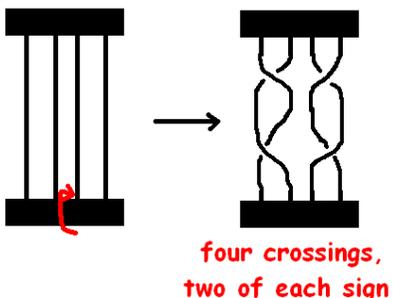
But look at the maneuvers we can perform on this system. There are essentially only four types of moves.

MOVE 1: *Push the bottom end of the felt through a slit to the left or to the right.*



We see that this move introduces six new crossings, all of the same sign. This means the index changes either by +6 or by -6.

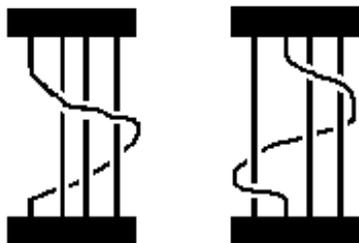
MOVE 2: *Push the bottom end of the felt through the middle slit.*



This move introduces four new crossings, but the total index does not change.

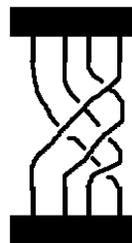
MOVE 3: *Pick up and move one strand around the base of the object.*

Here are two typical examples of this move.



We see that the total index again changes by ± 6 .

MOVE 4: *Rotate the bottom half of the felt half a turn.*

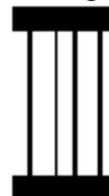


Such a move introduces six crossings all of the same sign and changes the index by +6 or by -6.

SO .. Is it possible to create the four-strand of index 4?



NO! We start with the figure of index zero:



and every move we perform on this object either keeps the index the same, or changes it by six. Whatever we create from this starting position must be a figure with index a multiple of six. An object with an index of four is thus unattainable.

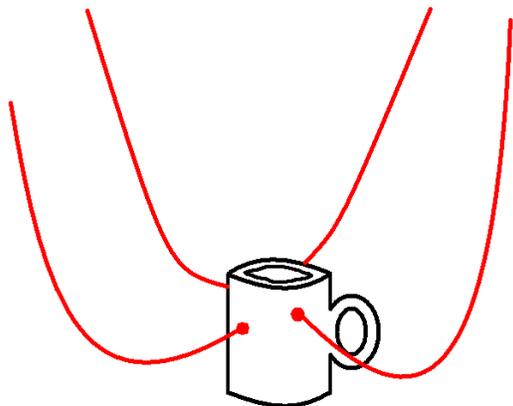
CHALLENGE: Does this mean that the following construct is attainable? (It has index -6.)



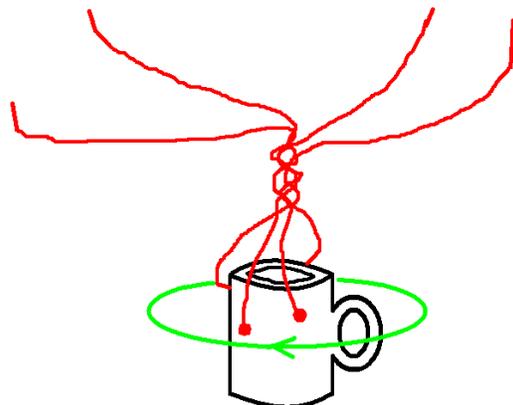
If so, attain it! If not, what this time is the mathematical obstruction?

THE TEA CUP CHALLENGE:

Hold a teacup up in the air in the center of a room and have friends tape four or five strings from the cup to various points about the room. Be sure to leave plenty of slack in the strings.



TASK 1: Rotate the teacup one full turn, 360° , tangling the strings in the process.



From this point on, the cup is to be held fixed in space, never to move again!

You and your friends' job is to now maneuver the strings around the cup and untangle them. Can you do it?

TASK 2: If you have trouble with the first task and want to give up... try giving the cup another full turn IN THE SAME DIRECTION (720°), tangling the strings even further! Again holding the cup fixed in space, maneuver the strings around the cup and untangle them. **This second task can definitely be done!** Absolutely try it!

CHALLENGE: Return to task 1 and try it again. If you seriously can't do it, perhaps come up with a mathematical proof of its impossibility. (Would a proof along the lines of impossibility of the four-braid apply?)

Comment: It would be mighty curious if it is impossible to untangle a tangle from one full rotation, but possible to untangle a doubly-worse tangle from two full rotations!

(Look up why Paul Dirac chose to assign half packets of quanta to angular momentum in the theory of quantum mechanics!)

THE OPENING PUZZLER:

WARNING: SPOILERS!

Fold your arms before you pick up the string!

RESEARCH CORNER:

Find some no-free-end four-strand braid patterns that can be made containing no twists within the individual strands. Develop a mathematical classification system of them. (Have we found all the "flat" no-free-end three braids?)

COMMENT: Sadly, this is the final essay for the 2012/2103 academic year. But worry not ... essays will resume in September. And check www.gdaymath.com regularly. I'll be posting more courses/experiences throughout the summer.

© 2013 James Tanton
stanton.math@gmail.com