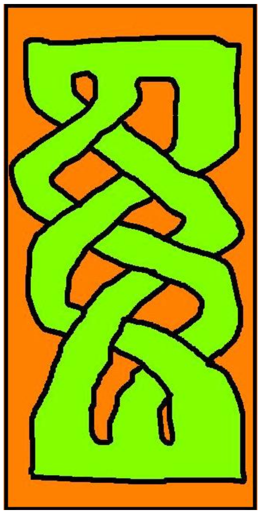

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WOAH! COOL MATH!

CURIOUS MATHEMATICS FOR FUN AND JOY

September 2017

THIS MONTH'S PUZZLER:

The majority of folk in the world does arithmetic in base ten. This is probably because of our human physiology: we have ten digits(!) on our hands and so we humans naturally think "ten" when it comes to matters of counting.

But there are folk in south-east Asia and elsewhere who find it natural, easy, and very human to count to 12 on just one hand.

a) *Lift up your left hand. Can you see how to naturally count out twelve using your thumb as a pointer?*

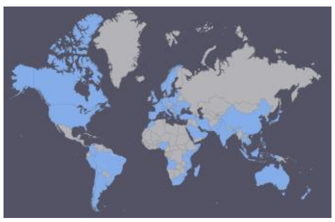
Base twelve arithmetic is actually a very handy–ha!–for matters of trade. The basic fractions one-half, one-third, and one-quarter often arise in the act of sharing goods, and "twelve" is mighty friendly number with respect to these fractions. "Ten" is not.

b) *Suppose a culture thinking base ten meets a culture thinking base twelve. If they combine their mathematics, what common base might they decide to work in?*

This video might be fun to watch. https://ed.ted.com/lessons/how-high-can-you-count-on-your-fingers-spoiler-much-higher-than-10-james-tanton)

GLOBALMATHPROJECT

Global Math Week is Oct 10-17 and it is already a global phenomenon! Tens of thousands of teachers and students, from over 75 countries, have signed on to do a joyous, uplifting, delightfully human, and delightfully compelling piece of mathematics together: the story of *Exploding Dots*.



Our site www.explodingdots.org and the FAQs there explain all. Taking part in Global Math Week is easy-peasy!

Experience *Exploding Dots* for yourself.
See the FAQs.

Register at our site.
Have you and your students count towards this global phenomenon!

Do some *Exploding Dots* with your students during Global Math Week.
One class period. Half a class period. Even 15-minutes will count! See the FAQs on our site to see how.

Share comments, photos, videos with the world on social media.
Be part of the global community.

Twitter: @globalmathproj
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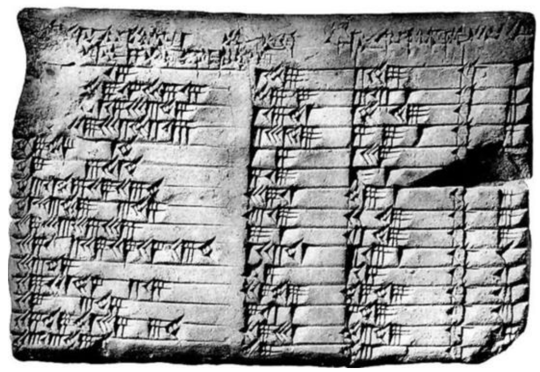
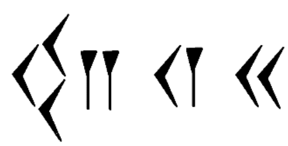
PLIMPTON 322

The Babylonians of 2000 BCE were adept mathematicians. We have records of their mathematics, the most famous being a 12.7 cm-by-8.8 cm fragment of a clay tablet called *Plimpton 322*. (It is item number 322 in the G. A. Plimpton collection at Columbia University.)



The fragment shows four columns of numbers written in their cuneiform script. It is a base sixty system—*sexagesimal*—with elements of base ten. Here the symbol  represents ten and the symbol  one, and thereafter place value in base sixty applies. For example,



reads as the "digits" 32, 11, and 20 and these digits are attached to powers of sixty. This number could be

$$32 \times 60^2 + 11 \times 60 + 20 = 115880.$$

But the Babylonians had no symbol for zero. (We make good use of zero in our notational system. For instance, it is clear to us that 23 and 203 and 20000300 are very different numbers.) The number presented could also be interpreted as

$$32\times 60^4+11\times 60^3+20\times 60$$
$$=417097200.$$

The Babylonians also had no symbol for the equivalent of a decimal point. So this number could also be

$$32\times 60+11\times 1+\frac{20}{60}=1931\frac{1}{3}.$$

The lack of zero and the lack of a sexagesimal point seems strange to our modern sensibilities. But the Babylonians apparently found context to be sufficient for defining which number they actually meant in any given situation.

Plimpton 322 shows four columns of numbers, fifteen rows in all. The rightmost column simply gives the row number. It is the first three columns that are of interest. In modern, base-ten notation the numbers we see there are as follows.

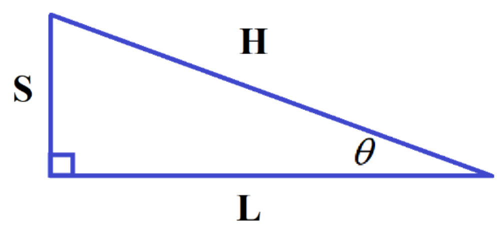
(1).9834026	**119**	**169**
(1).9491586	**3367**	**4825**
(1).0188025	**4601**	**6646**
(1).8862479	**12709**	**18541**
(1).8150077	**65**	**97**
(1).7851929	**319**	**481**
(1).7199837	**2291**	**3541**
(1).6927094	**799**	**1249**
(1).6426694	**481**	**769**
(1).5861226	**4961**	**8161**
(1).5625000	**45**	**75**
(1).4894168	**1679**	**2929**
(1).4500174	**161**	**289**
(1).4302388	**1771**	**3229**
(1).3871605	**56**	**106**

(Sometimes the numbers in the first column contain a leading 1, and other times they don't.)

I personally recognize 119 and 169 as part of a Pythagorean triple: $119^2+120^2=169^2$. In fact, each pair of integers we see are part of a triple $S^2+L^2=H^2$.

	S	**H**	**L**	**theta**
(1).9834026	**119**	**169**	**120**	**44.76°**
(1).9491586	**3367**	**4825**	**3456**	**44.25°**
(1).0188025	**4601**	**6646**	**4800**	**43.79°**
(1).8862479	**12709**	**18541**	**13500**	**43.27°**
(1).8150077	**65**	**97**	**72**	**42.08°**
(1).7851929	**319**	**481**	**360**	**41.54°**
(1).7199837	**2291**	**3541**	**2700**	**40.32°**
(1).6927094	**799**	**1249**	**960**	**39.77°**
(1).6426694	**481**	**769**	**600**	**38.77°**
(1).5861226	**4961**	**8161**	**6480**	**37.44°**
(1).5625000	**45**	**75**	**60**	**36.87°**
(1).4894168	**1679**	**2929**	**2400**	**34.96°**
(1).4500174	**161**	**289**	**240**	**33.86°**
(1).4302388	**1771**	**3229**	**2700**	**33.26°**
(1).3871605	**56**	**106**	**90**	**31.90°**

Mathematical historians, of course, noticed this too. These fifteen rows are values of Pythagorean triples (the smallest and the largest values) from right triangles with angles θ steadily decreasing.



And the numbers in the first column are either the values of $\tan^2\theta=\left(\frac{S}{L}\right)^2$ or $1+\tan^2\theta=\left(\frac{H}{L}\right)^2$.

WHAT'S "COOL" HERE?

Look at the triples the Babylonians listed. Each middle L value for the triples they chose turns out to be composed only of the primes 2, 3, and 5.

$$120=2^3\cdot 3\cdot 5$$
$$3456=2^3\cdot 3^3$$
$$4800=2^6\cdot 3\cdot 5^2$$
$$\vdots$$

This means that the reciprocals of each of these numbers is a "finite decimal" in base 60.

For example,

$$\frac{1}{120} = \frac{30}{60^2} =$$

and

$$\frac{1}{3456} = \frac{1}{2^6 \cdot 3 \cdot 5} = \frac{45}{60^3} =$$

,

and so the Babylonians were able to express each of the values $\tan^2\theta$ and $1+\tan^2\theta = \sec^2\theta$ as exact finite sexagesimals. Moreover, the Babylonians had good methods for accurately approximating the square roots of values. This meant they could approximate values of $\tan\theta$ to any desired degree of accuracy from the exact values of $\tan^2\theta$.

So Plimpton 322 seems to be a reference table for computing the slopes of different ramps that vary in angle of elevation by, more-or-less, a steady rate from 32° to 45°. And this seems to be the conclusion of the D.F.Mansfield and N.J. Wildberger's latest paper, August 2017, "Plimpton 322 is Babylonian exact sexagesimal trigonometry," http://www.sciencedirect.com/science/article/pii/S0315086017300691.

RESEARCH CORNER

Have a look at Mansfield and Wildberger's paper and the press in response to it. See, for example, the video in *The Guardian* piece https://www.theguardian.com/science/2017/aug/24/mathematical-secrets-of-ancient-tablet-unlocked-after-nearly-a-century-of-study.

Do you agree with what you read and see?

Why do you think the Babylonians chose more complicated triples over simpler ones? For example, row 11 of the table has the triple $(45,\ 60,\ 75)$, which is just the standard $(3,4,5)$ triple multiplied through by 15.

Euclid, some 1500 years later, proved that every primitive Pythagorean triple (S,L,H) can be produced by choosing a pair of relatively prime numbers m and n and setting $S = m^2 - n^2$, $L = 2mn$, and $H = m^2 + n^2$. (Even if m and n share a common factor, this procedure produces a Pythagorean triple.) Is every triple in Plimpton 322 so produced? If so, is there anything special about the values m and n used?

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