EXPLORATION 2

FACTORS AND PRIMES

Open doors and closed doors. Learn how locker doors reveal patterns in factors of numbers.

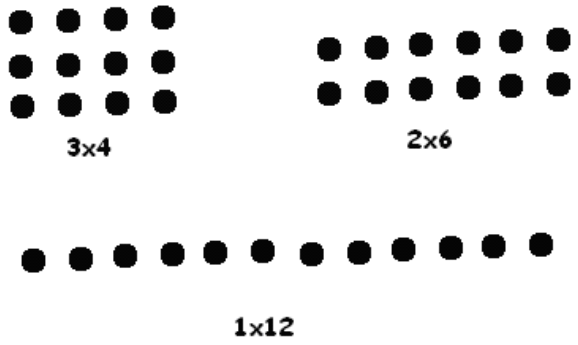
TOPICS COVERED: Factors of Numbers. Composite Numbers numbers.
The great Locker Experiment.

A. GETTING STARTED

Let's begin with a question:

How many different rectangles can one produce with 12 dots?

We have three (assuming that the 90-degree rotation of a rectangle is considered the same rectangle: ▭ = ▯).



The numbers that arise as the widths and lengths of these rectangles – 1, 2, 3, 4, 6, and 12 – are called the factors of the number 12. They are also the numbers that arise in products that give 12:

$$12 = 3 \times 4$$
$$12 = 6 \times 2$$
$$12 = 1 \times 12$$
$$12 = 2 \times 2 \times 1 \times 3$$

Question 1:
a) List all the factors of 18.
b) List all the factors of 30.
c) List all the factors of 600.
d) List all the factors of 13.
e) List all the factors of the number 1.

Question 2: How many different rectangles can one make with 18 dots? With 13 dots? With 600 dots? (Again assume that a 90° of a rectangle is considered the same rectangle.)

In general, within the realm of positive counting numbers:

> A number a is called a factor N if we can write $N = ab$ for some number b.

Question 3: Is this definition confusing? Jenny says that "3" might not be a factor of nine because $9 = 3 \times 3$, meaning that *a* and *b* are the same number in this example. She thinks this is a problem. Is it?

Jargon: People like to categorize numbers:

> A number is called composite if it has more than two factors.
> A number is called prime if it has *exactly* two factors.

For example, 18 is composite because it has six factors (1, 2, 3, 6, 9, 18). The number 13 is prime because it has exactly two factors (1 and 13).

Comment: According to this definition, the number 1 is neither prime nor composite. (The number 1 has only one factor!)

ASIDE: Many students and teachers are deeply confused about this and think that 1 should be prime. The problem is that people will often say that a prime number is a number "whose factors are only 1 and itself."

Question 4: According this dubious definition, does "1" fit the bill of being prime?

Mathematicians really don't want 1 to be considered a prime number for the sake of factoring. For example, the number 100 can be written as a product of smaller numbers:

$$100 = 4 \times 25$$

which can in turn be written in terms of smaller product still:

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