PHILOSOPHY OF THIS SITE

PARCEL ONE: Do the words organic and artistic come to mind when you think "mathematics"? How about deep and rich? Maybe beautiful? Joyful? Perhaps that is asking for too much.

Let's try … Safe? Logical? Useful? (Dry?)

As a teacher, tell me honestly when you used the quadratic formula in your everyday life. Or that the product of two negative numbers is positive. Or that there are actually three cube roots of the number eight. Have you met a carpenter who actually uses sines and cosines in her work?

Again be honest … Do you personally understand how math is useful? Do you know how math is used in your DVD player? To predict tomorrow's forecast? To analyse seismic waves? What is the math that you actually engage in and can personally attest as useful: Balancing numbers on a tax form? Converting measurements in cooking? Number crunching? Is that the honest answer? And is that math?

SO … what are we as math teachers teaching? And for what end?

PARCEL TWO: It seems I am leading to a bleak conclusion, but that is not at all the case. I am heading instead to something joyful, exhilarating and, today, radical. Why not just teach math for what math is? Why not let math fully and unashamedly speak for itself?

Math is the beautiful, rich, joyful, playful, surprising, frustrating, humbling and creative art that speaks to something transcendental. It is worthy of much exploration and examination because it is intrinsically beautiful, nothing more to say. Why play the violin? Because it is beautiful! Why engage in math, because it too is beautiful!

PARCEL THREE: Mathematics is thought of, in some unspecified way, as "practical" and "useful" by the majority of folk on this planet, and they associate with it numbers and calculations. Mathematicians find the question:

What is 1345×7189 *?*

uninteresting. But the question "

Why should 1345×7189 *and* 7189×1345 *give the same answer?*

a much richer conundrum that homes in on a truth about numbers. Or further:

Is the commutative law true for all types of numbers?

This is much more profound a question! Or better yet:

Can we create a system of arithmetic for which the commutative law is definitely NOT true?!

How wickedly fun is that!

Why? and what if? questions over simple what? questions are the much deeper, much more playful, and, in the end, much more meaningful.

PARCEL FOUR: There is no doubt that mathematics possesses immeasurable utility (and in fact a considerable bulk of mathematics is inspired and motivated by "real world" practical problems), but the mathematics that comes of it soon begins to speak to something deeper. Mankind has not engaged in mathematics for thousands of years simply because it is useful (why did the Babylonians compute hundreds of Pythagorean triples?), but because it speaks to something transcendental.

Sure, we must teach students certain skill sets so as we don't keep pausing over trivial matters on the way to deeper ones, but that set of practical skills really is minimal. Mathematical matters of everyday life are probably sufficiently dealt with by early middle school. In the high-school realm, however, with practical skills essentially out of the way, we can work on issues of intellectual maturity and growth, on depth of understanding and of flexibility of thought, on learning how ask questions, to create questions and to extend and push boundaries. These are the skills required for true success with innovation in business and breakthroughs in scientific research. But what has happened? High school curricula head deeper into the direction of skills, usually in a manner that can only be described as oppressive.

> HIGH SCHOOL: Memorise the quadratic formula, the vertex formula, the double angle formulae, the translation properties of graphs, the surface areas of spheres, cones and cylinders, the box method for graphing hyperbolas, the change of base formula for logarithms, the properties of a parallelogram, trapezoid, and a kite, the compound interest formula, the nth term of an general arithmetic sequence, and so on and so on.

HIGH SCHOOL: Don't ask: Why is an ellipse called a "conic section"? Don't explore sections of shapes other than cones. Don't ask if raising numbers to second and third powers actually has anything to do with squares and cubes? (What about raising numbers to a fourth power? Why don't we have a word for that?) Don't ask if there are any numbers beyond the complex numbers? Don't ask if we can FILO instead of FOIL? Don't ask why *quadratics* have a name associated with the number four? Don't ask about 0 raised to the zeroth power or try to come up with a multitude of permissible values for it. And don't ever ask at any time for any topic: Why do we need to know this? (What do you say in response to this question?)

HIGH SCHOOL: Memorise the jargon first. Give names to things before you know they could be otherwise. (As an example of the absurdity of this, can you name something that doesn't satisfy the reflexive property? Can you give me an example of an instance where transitivity fails? It is only when one has an instance of something that doesn't work need we give a name to the phenomenon that does work. For example, why have the word "tall" if there is no concept of shortness?)

HIGH SCHOOL: Do this weird set of tasks now because "they will be useful later." (Why on earth, exactly, would any beginning algebra and functions student actually want to simplify $\frac{f(x+h)-f(x)}{h}$?)

THE GOAL OF THIS SITE

Given the constraints and demands of state-wide and national testing placed on the subject today, it is not at all clear how to bring the full joyful spirit of mathematics into the high school curriculum.

In this testing environment one has to deal a sad array of sillinesses: The perfectly acceptable mathematical answer $\frac{1}{\sqrt{2}}$ is rejected over the allegedly "simplified" $\frac{\sqrt{2}}{2}$; the properties of 30-60-90 triangles are tested over and over again although very few mathematicians could recite you their side ratios and very few students realize that each is just half an equilateral triangle; joyless two-column proofs abound even though no mathematician ever writes a proof that way!

But I want to take baby steps to make a change. That is why I left college teaching to become a high-school teacher.

On this site I present a whole host of curriculum essays, more general mathematical essays, and puzzlers. I offer books that cover the content of the high school curriculum in a joyful and accessible way (with no sacrifice to rigor) respectful of the beauty and wondrous creative nature of mathematics. I show how even high-school mathematics can astound and delight, and how clean and uncluttered key topics can be.

I seek clarity of understanding and freedom to think. I encourage the push of boundaries and the willingness to explore. There is absolutely no shying away from the *whys* and *what ifs*.

I want teachers and students to like mathematics, to understand mathematics and to develop a sense of personal curiosity.

I want teachers and students to have fun and enjoy and appreciate and admire and be willing to explore and play.

My baby steps towards these goals are the following:

STEP 1: Remove unnecessary clutter.

Even if the mathematics of quadratics is not particularly exciting, there is no need to make it joyless! Understanding supersedes any need for memorizing. Perhaps students - and teachers – prefer to memorize formulas for a sense of "safety." But memorization is far from lasting and does not speak to a sense of perspective and understanding. It doesn't provide a sturdy platform for play and investigation. And it generally only allows you to answer questions others have already provided for you (and have already answered in the back of the book!)

Let's let go of the intellectual clutter of memorizing. If formulas happen to stick in your mind, fine, but let's not make that the goal.

And let's let go of volumes of jargon! Only if a unique idea or situation presents itself repeatedly we might want to give it a name. If it doesn't, don't bother naming it! (And certainly don't name something for the sake of naming it and nothing more!)

Here are three forms of an equation of a line:

$$y = mx + b \qquad y - a = m(x - b) \qquad Ax + By = C$$

These all have names (which, I personally, cannot recall).
Here are some more forms of equations of lines:

$$\frac{A}{y-p} = \frac{B}{x-q} \qquad y^2 = ax^2 \qquad \lambda = \frac{A-x}{B-y}$$

These don't have names. Hmmm. Maybe we need jargon for these too!

I'd rather have a student do something interesting with a quadratic than to rearrange it in "vertex form" (whatever that is!), or have a student simply rearrange an equation in some (unnamed) way of her choosing so that she can see an easier why to address the question at hand.

STEP 2: Keep it simple

Mathematics at its best is simple. Mathematicians call this *elegance*. Seeing to the core of a set of ideas reduces their study to a few key principles. And understanding those key principles allows everything to fall into place.

Realising, for example, that a function of the form $y = ax^2 + bx + c$ is just a transformed version of the U-shaped graph $y = x^2$, makes graphing quadratics straightforward. (No need to memorise a vertex formula!) Realising that multiplication by *i* has a geometric effect of a 90-degree rotation on points in the plane makes playing with complex numbers easy and fun. Knowing that "sine" arose as the height of a star at a certain angle makes trigonometry (which really should be called "circle-ometry") elegant and beautiful. And this business of "is order relevant?" is unnecessary clutter: there is no fundamental difference between a permutation and a combination!

The curriculum essays offered on this website home right in on the core ideas at hand and bring simplicity of thought to particular high-school topics. The texts available from this site do the same.

STEP 3: Let the math speak for itself

One of the greatest surprises – one that truly shocked and delighted mathematicians - is a simple formula that unites trigonometry and complex numbers, simplifying all of trigonometry in one fell swoop. Through a study of slope one comes to realize that e^{ix} is $\cos x + i\sin x$ in disguise. So often I have seen teachers introduce this formula in an honors pre-calculus class as a "useful" formula for computing complicated quantities without any hint from where this formula might come or any hint as to why it might be true. It becomes a dry result accompanied with 50 practice problems, some of which will be on the test. What has happened to beauty, to surprise, to shock? The great Leonhard Euler was dumbfounded by this result. So should be every person when she first encounters it!

So … if we insist on teaching this "useful formula" in a pre-calculus course, then let's let it be a shock. Let's talk about slope and shape a path of exploration that allows this formula to inevitably fall into place just using the tools and techniques at hand to the students. Let it creep up from behind and grab the intellect by surprise! And let students enjoy the experience of surprise. The materials available from this site show how such a path could be.

And there are many places in the curriculum (even in standard non-honors courses) where we can naturally let the mathematics speak for itself. So why don't we? Beginning geometry speaks of "parallelism" yet it is interesting to note that Euclid, the founder of geometry as we know it, never used the word "parallel" in his work. Euclid deliberately avoided the term as a concept that is beyond human. (Can we ever possibly check whether or not two lines are actually parallel?) He worked instead on finding a local concept that would get at the heart of parallelism without involving an infinite concept. Clever fellow! Why don't we speak of this and explore this with our students?

John Napier set out to help his scientific colleagues of the day by inventing a means to convert multiplication problems, which are very hard to conduct by hand, to simpler addition problems. His approach turned out to be very complicated in theory, but easy to implement in practice, and he called his tools "logarithms." It wasn't until 100 years later than mathematicians realized that logarithms are a very simple concept in disguise! The math is elegant and straightforward, but the complicated name remains. (And a complicated approach to teaching them remains as well.)

So … Simply put, let's let the math shine through.

The essays and materials included on this site show how.

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