Some Clarity on the
COMMON CORE STATE STANDARDS IN MATHEMATICS
and "ADVANCED ALGEBRA."

Response to THE MATH MYTH by James Tanton

WHAT THE COMMON CORE IS

The Common Core State Standards in Mathematics is simply two lists. (Download them! www.corestandards.org/Math/ .)

One is a list of *Content Standards*, all the pieces of mathematics agreed upon students should see during their thirteen years for schooling, from grade K to grade 12.

There is nothing really new here – it covers essentially the same content we have been teaching for decades – just arranged, at long last, very carefully to be present a pedagogically and age appropriate set of storylines of mathematics ideas. Very traditional mathematics is still present: the long division algorithm, for example, appears in full form in grade 6, after students have developed sufficient number facility in previous grades to understand what the algorithm is doing, not to just to perform it in a rote fashion for the sake of getting answers. (If getting an answer is the only goal, then we are each better off pulling out our smart phone.)

The other list is new. It is a list of eight, just eight, *Mathematical Practice Standards* asking that we properly attend to the thinking, doing, and good communication of mathematics.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

(Look at the downloaded document for detailed explanation of each of these statements.)

My enthusiasm for the Common Core is really based on these eight standards. Who can object to attending to thinking?

One can learn about the historical development of the Common Core State Standards in the downloaded document, further, here, for example: http://www.usnews.com/news/special-reports/articles/2014/02/27/the-history-of-common-core-state-standards .
A first call to consider having common State standards was made in 2006/2007. The work to develop them occurred, of course, during the years after this.

SOME THINGS THE COMMON CORE IS NOT

Many have the impression that the Common Core consists of over a thousand new standards. Dr. Hacker does too, writing on page 121 of *The Math Myth* "…which by my count totaled **1,386** new "standards"" (I am not sure why the word *standards* appears in inverted commas).

I personally counted a total of **436** mathematics content standards, even counting any itemized sections of a single standard as separate. I know I have likely miscounted, but I am not off by a count of 850.

K-8: **287** standards
HS: **149** standards, **30** of which are optional.

It is too easy to confuse State's choices in implementing the Common Core as the Common Core, in particular, with the curriculum each State chooses to adopt as, they believe, is appropriately aligned with the Common Core.

But note that the Common Core itself is not a curriculum. There are no "unbending lesson plans" (page 118), no "uniform tests and parallel scoring systems" (page 16), as Dr. Hacker claims in his book, and the standards, as written, are not questions for pupils to confront (page 5).

In fact, the standards are not written for student consumption at all. They are presented in language appropriate for educators and curriculum writers, succinctly conveying the concepts to be translated for student classroom activity.

As an example, Dr. Hacker mentions, with alarm, on page 5 that the "associative property" is called for. This term is used, for instance, in a grade 1 standard which calls for students to notice the associative property of addition. But it is not calling for students to use or even hear that term, just to notice that 1+3+6, for instance, makes ten no matter in which order one performs the additions. (Follow 4+6 or 1+9, both give ten. How delightful!)

Many believe that the Common Core State Standards are full of meaningless algorithms, irrelevant for those in the workforce. (Page 42.) The word *algorithm* appears in just five of the content standards, each in grades 3-6, refering to the long addition, subtraction, multiplication, and division algorithms. (True, those in the workforce probably use their smart phones for multidigit arithmetic.)

A Curious Exercise: Look at the State Standards for those states that have rejected the Common Core. Seriously, how different are they from the Common Core State Standards?

WHAT IS MEANT BY "ADVANCED ALGEBRA"?

It is hard to ascertain what Dr. Hacker means exactly by "advanced algebra," as he calls it in his book. There are certainly a number of (actual and pseudo) mathematical terms he seems to imply belong to the algebra II curriculum as dictated by the Common Core State Standards – azimuth (chapters 1, 10, 12), ellipsoidal coordinates (chapter 11), reentrant angles (chapter 1), geometric paraboloids (chapter 2), line multiple representations of exponential models (chapter 11), to name a few. (One can check that not one of these terms appear in the document by using the "control F" search feature.)

He writes about algebra and trigonometry being too abstract and too advanced to be suitable as career preparation, yet praises the Northeastern Mississippi Community College course *Machine Tool Mathematics* which focuses on “algebraic and trigonometric operations essential for machining” (page 42).

In chapter 8 he extols the cross-disciplinary work of the Hawken School and writes “The Hawken School, in Cleveland, Ohio, for example, takes pride in its interdisciplinary programs. “…There is no way, it says, the Common Core can evaluate offerings like that” (page 127). Yet, go to the Hawken School’s webpage and look up their high-school mathematics curriculum and you see algebra I, geometry, algebra II, followed by lists of advanced courses, with all three of the basic courses required of all students for graduation. Moreover, look closely at the course descriptions and you see listed the very topics that Dr. Hacker worries about. (These topics actually fit with the Common Core State Standards – and they go a bit beyond. So maybe the message here is that even progressive schools could deem “standards” topics as relevant, good, and appropriate, even for cross-disciplinary work?) My apologies for calling out this wonderful school in this piece. I am on the side of the school here.

SCARY MATH TERMS

Here is a list of (pseudo and actual) math terms that appear in *The Math Myth* -- parentheses give chapter numbers -- that Dr. Hacker seems to imply appear in the Common Core.

<u>**TERMS THAT DON’T APPEAR AT ALL**</u>

Azimuth (1,10,12)
Reentrant angles (1)
Radical notations (1)
Elliptical equations (1)
Parabolic geometry (1)
Prime Factorization (1)
Squared binomials (1)
Trinomial (2,3)
Algebraic vectors (2)
Geometric paraboloids (2)
Ellipsoidal coordinates (6)
x-Equation (1)
Line multiple representations of exponential models (11)

Re “radical notations”: The word **radical** appears in <u>three</u> standards (N.RN.1, N-RN.2, A.REI.1). We are refering to square roots, cube roots, and the like.

Re “elliptical equations”: The word **ellipse** appears in <u>one</u> optional standard (G.GPE.3(+)).

Re “algebraic vectors”: The word **vector** appears in the cluster of standards N-VM, all optional, attending to matrices and vectors.

<u>**OTHER WORDS**</u>

The word **ASYMPTOTE** (1,10,11,12):
It appears in <u>one</u> optional standard F.IF.d(+)

The word **ALGORITHM** (3):
“… the algorithms taught in school are often not the computational methods of choice for workers.” (Page 42)

The word appears in <u>five</u> standards only, in grades 3-6. The algorithms being discussed are base ten addition, subtraction, multiplication, and division. (3.NBT.2, 4.NBT.4, 5.NBT.5, 6.NS.2, 6.NS.3)

The term **PYTHAGOREAN TRIPLES** (1,8,11):
Appears in <u>one</u> standard as an an example of a possible application.

The term **PASCAL’S TRIANGE** (1,8,11):
Appears <u>one</u> optional standard (A.APR.5(+)).

TERMS THAT ARE IN THE COMMON CORE

RATIONAL EXPONENTS (2,11):
Appears in <u>two</u> standards(N.RN.1, N.RN.2).

LINEAR INEQUALITIES (2):
Appears in <u>one</u> standard (A.REI.12).

ASSOCIATIVE PROPERTIES (1):
Appears in <u>five</u> standards, one optional (1.A.3; 3.A.5; 5.MD.5a; N.CN.2; N.VM.9(+)).

INVERSE FUNCTIONS (2):
Appears in <u>five</u> standards, all optional (F.BF.4b,c,d(+); F.BF.5(+); F.TF.7(+)).

COMPLEX NUMBERS (2):
Appears in <u>eight</u> standards, five of which optional (N.CN.1, 2; N.CN.3,4,5,6,7,8(+)).

IRRATIONAL NUMBER (1):
Appears in <u>four</u> standards (8.NS.1,2; 8.EE.2; N.RN.3).

COSINE (2):
Appears a number of times in the trigonometry standards.

LINEAR AND QUADRATIC EQUATIONS (2):
Appear multiple times.

CORRELATION versus CAUSATION

"When A and B are observed together, it's often wise to look for a C that may have caused *both* A and B. Yet this simple fact is seldom taught in mathematics classes. (Probability may be incuded, but causality typically isn't.)" (Page 59)

Look at standard S-ID.9: *Distinguish between correlation and causation.*

PROOFS

"The proofs associated with mathematics are schematically structured, with each step numbered or similarly designated."
(Page 83)
The two-column proofs of yore appear nowhere in the Common Core.

Aside: A SHOCKER FOR MANY EDUCATORS

The word **simplify** appears nowhere in the Common Core!

IS "NUMERACY 101" IN THE COMMON CORE?

On page 42, Dr. Hacker presents a quote from Lynn Arthur Steen: "What current and prospective employees lack is not calculus or college algebra, but a plethora of more basic quantitative skills that could be taught in high school, but are not."

One can check that Dr. Steen wrote this in 2003, years before the conception of the Common Core. It is not thus not criticism of the Common Core, but of the state of matters before it. And we can see that the Common Core has worked to address this very concern with Mathematical Practice Standard number 2:

MP2: *Reason abstractly and <u>quantitatively</u>.*

Also, **Illustrative Mathematics,** www.illustrativemathematics.org, established in 2011, is a community of educators and founders of the Common Core working to illustrate the context and meaning of each and every content standard with a concrete student-ready example. Teachers can incorporate these examples directly in their lessons. We see among them plenty of examples dealing with all the quantitative skills Dr. Hacker calls for:

Agility with numbers

Estimation and context of numbers

Comparing percentages, developing intuitive understanding

Spatial reasoning

Approximation – playing with irregular areas, approximating pi, making sense of mathematical processes.

There are written curricula that have incorporated this work, **Eureka Math** (http://greatminds.net/maps/math/home) for example, is a prime example.

ON UNDERSTANDING PI

> "… proved the reliability of pi, or if it was just another clue in a mystery we may never solve." (Page 195)

The concept of pi is indeed subtle, and Dr. Hacker works to help students develop good intuitive understanding of role of the number in mathematics with the activities he describes on pages 193 and 194. Very similar activities already appear in many curricula.

> "… nature also has some numbers that control or explain how the world works. One of them is π, whose 3.14159 goes on infinitely, at least as far as we know."
> (Page 187)

In 1761, Johann Heinrich Lambert settled this question once and for all and proved that pi is an irrational number, and so has an infinitely long decimal expansion possessing no repeating pattern. (And later, in 1882, Ferdinand von Lindemann proved that pi is actually transcendental). There is a wonderful story of mankind's wondering, for millennia, whether or not pi is a rational and if the classic problem of *Squaring the Circle* could ever be solved. Lambert settled it once and for all. A good curriculum will not deny students this lovely story.

> "Pi's formula for cylinders is $\pi r^2 h$."
> (Page 194)

Mathematical Practice Standard 6: *Attend to precision* is actually referring to precision of language.

TEST QUESTIONS

I too am deeply concerned about the effect of standardized testing has on mathematics classroom culture, especially unenlightened testing that reinforces the attitude that math is "about getting the right answers," and the speedier the better.

But there are the occasional standardized test questions that do attend to meta thinking, and we should point that out. They are really testing whether or not a student can "cut through the clutter" and see the bigger picture of what is going on.

Unfortunately, Dr. Hacker missed this point on two examples he presents.

Consider this question from page 42:

Two charges (+q and –q) each with mass 9.11 x 10^{31} kg are placed 0.5 m apart and the gravitational force (F_g) and electric force (F_e) are measured. If the ratio F_g/ F_e is 1.12 x 10-77, what is the new ratio if the distance between the charges is halved?

Five numerical options to choose from are offered.

(The two typos in this question appear in the question as presented in the book.)

The question does indeed look scary.
But the point of this question is to see if students can "step back" from matters and realize that the forces discussed in their physics course vary with distance in identical matters, and so the ratio of forces does not change.

On page 90 Dr. Hacker presents this test question:

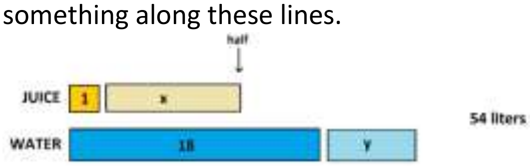
For a certain board game, two dice are thrown to determine the number of spaces to move. One player throws the two die and *the same number comes up on each of the dice. What is the probability that the sum of the two numbers is 9?*

Again. This is a question asking students to “step back” from memorized procedures to see if they can “see through” matters. The answer is clearly zero as two identical numbers on dice will never add to 9.

But there are indeed many examples of joyless, impractical questions to be found in the myriad of state and national tests of the past decade. For example, on page 74 Dr. Hacker presents

A 19-liter mixture consists of 1 part juice to 18 parts water. If x liters of juice and y liters of water are added to this mixture to make a 54-liter mixture consisting by volume of 1 part juice to 2 parts water, what is the value of x?

If I were required to present my students with such a question, I’d have the goal be to draw a diagram that represents the problem. I’d hope they would sketch something along these lines.



The picture clarifies matters, its exemplifies lovely mathematical thinking, it cuts through the complication of a joyless algebra grind, and shows that $1+x$ must be one of the three parts that make 54 liters.

I have been told that considerable work is being done to develop online testing and online grading that allows for free written responses and probes for demonstration of understanding and intellectual flexibility. Let’s hope, if standardized testing is going to be a fixture in our teaching culture that it can indeed move deeply in this direction. (But let’s please work to let go of this damaging demand for speed doing.)

MANDARINS

In chapter 7, and throughout his book, Dr. Hacker points to some kind of perturbing influence mathematics “mandarins” exert on the shape and conduct of K-12 mathematics education. I admit I struggle to comprehend this point. (Perhaps I am one of the mandarins?) Let’s please check the claims made and the alleged supporting examples. (In particular, the case presented on pages 53 and 54.)